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Technical Report

**Satellite Earth Stations and Systems (SES);
Guidelines for determining the parts of
satellite earth station antenna radiation patterns
concerned by the geostationary satellite orbit protection**

Reference

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Keywords

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Foreword

This Technical Report (TR) has been produced by ETSI Technical Committee Satellite Earth Stations and Systems (SES).

The present document is intended to be used as a guideline to assist with the measurement methods of the compliance with the off-axis EIRP density limits for the protection of the GSO arc specified within the Harmonized standards applicable to satellite earth stations. The present document is intended to be cited in the harmonized standards as a non normative document, with its reference listed in the Bibliography annex.

Introduction

Most of the harmonized European standards (ENs) applicable to satellite Earth Stations (ESs), prepared in ETSI by the TC SES, contain off-axis e.i.r.p density limits for the protection of other satellites on the Geostationary Satellite Orbit (GSO) and in its vicinity: within ±3° north/south. These limits are based on the relevant ITU-R Recommendations and Regulations such as ITU-R Recommendations S.728-1 [1], S.580-6 [2], S.465-5 [3], S.524-8 [4], ITU-R Resolution 902 [5]. For non-symmetrical antenna patterns around their main beam axis, the requirement for the protection of the GSO arc may be limited to the off-axis directions towards the visible part of GSO arc. For verification of the conformance with the applicable EN of earth stations fitted with antennas with non-symmetrical patterns, it is necessary to provide guidelines for determining the parts of satellite earth station antenna radiation patterns concerned by the geostationary satellite orbit protection.

The purpose of the present document is to provide a method for the determination of the range of off-axis directions which could be oriented towards the visible part of GSO according to the following operational parameters:

1) the range of operational latitudes of the ES;

2) the minimum antenna main beam axis elevation;

3) the type of antenna mount used (e.g. azimuth-elevation, equatorial);

4) the alignment accuracy of the antenna mount axes;

5) the minimum offset angle, relative to the satellite position, on the GSO from which protection of the GSO arc is required.

Within the present document:

- The GSO arcs is presented in various coordinate systems, e.g. geocentric for an observer far from the Earth, tropocentric for an observer located at the earth station, in a Cartesian coordinate system defined on the antenna for an observer located at the antenna flange, in a polar coordinate system defined on the antenna which will be use to determine the plane containing the antenna main beam axis, concerned by the GSO protection.
- The contours of the mapping on the antenna radiation patterns where the shadow of the GSO arcs may occur, for various type of antenna mount.
- The coordinate system used within the present document (see figure 6) may be different from other antenna coordinate systems. In particular, the present document uses the x-axis as the antenna main beam axis instead of the z-axis.
- The effect of the alignment error of the antenna mount axes on the contours of theses mappings.
- The mathematical analysis used for the determination of the GSO shadow and the contours of the mappings.
- The description of a mathematical method for the determination of the contours.
- A presentation of the Excel tool implementing the mathematical method and provided with the present document.

The Excel Tool provided with the present document has been developed on the base of the present document and may be used to obtain indicative results, but in any case neither ETSI, nor the ETSI technical committee members who prepared and approved the present document and the tool are responsible of the errors which may remains nor for the direct or indirect consequences of those errors.

1 Scope

The present document provides a method for the determination of the range of off-axis directions which could be oriented towards the visible part of GSO according to the following operational parameters:

1) the range of operational latitudes of the ES;
2) the minimum antenna main beam axis elevation;
3) the type of antenna mount used (e.g. azimuth-elevation, equatorial);
4) the alignment accuracy of the antenna mount axes;
5) the minimum East-West offset angle, relative to the satellite position, on the GSO from which protection of the GSO arc is required; and
6) in the case of an antenna designed for operation with a specific list of satellites, the minimum distance of the antenna to the sub-satellite points at the surface of the Earth.

These operational parameters are either:

- specified within the standard (e.g. the minimum antenna main beam axis elevation is equal to 7°); or
- declared by the applicant; or
- indicated within the user documentation.

2 References

For the purposes of this Technical Report (TR), the following references apply:

[1] ITU-R Recommendation S.728-1: "Maximum permissible level of off-axis e.i.r.p density from very small aperture terminals (VSATs)".

[2] ITU-R Recommendation S.580-6: "Radiation diagrams for use as design objectives for antennas of earth stations operating with geostationary satellites".

[3] ITU-R Recommendation S.465-5: "Reference earth-station radiation pattern for use in coordination and interference assessment in the frequency range from 2 to about 30 GHz".

[4] ITU-R Recommendation S.524-8: "Maximum permissible levels of off-axis e.i.r.p density from earth stations in geostationary-satellite orbit networks operating in the fixed-satellite service transmitting in the 6 GHz, 13 GHz, 14 GHz and 30 GHz frequency bands".

[5] ITU-R Resolution 902 (WRC-03): "Provisions relating to earth stations located on board vessels which operate in fixed-satellite service networks in the uplink bands 5 925-6 425 MHz and 14-14,5 GHz".

3 Definitions, symbols and abbreviations

3.1 Definitions

For the purposes of the present document, the following terms and definitions apply:

antenna azimuth axis (Az-axis): direction towards the left hand side and parallel to the horizontal plane when the antenna is oriented towards the South

antenna elevation axis (El-axis): direction parallel to the local vertical when the antenna is oriented towards the South with its main beam axis horizontal

antenna main beam axis: direction where the antenna gain is maximum

applicant: manufacturer or his authorized representative within the European Community or the person responsible for placing the apparatus on the market

azimuth: angle of the projection of the considered direction on the local horizontal plane with the local North direction

elevation: angle of the considered direction with the local horizontal plane

GSO vicinity: band within 3° of the GSO on the sphere of radius ρ = 42 164 km around the Earth

3.2 Symbols

For the purposes of the present document, the following symbols apply:

rad	radian
δi	the antenna azimuth axis inclination variation or equivalent value

3.3 Abbreviations

For the purposes of the present document, the following abbreviations apply:

Az-El	Azimuth-Elevation
e.i.r.p	Equivalent Isotropicaly Radiated Power
EN	European standard (Norm)
ES	Earth Station
ESV	Earth Station on board a Vessel
FSS	Fixed Satellite Service
GPS	Global Positioning System
GSO	Geostationary Satellite Orbit
ITU	International Telecommunication Union
ITU-R	ITU - Radio sector
SES	Satellite Earth station and Systems
TC	Technical Committee
VSAT	Very Small Aperture Terminal
WRC	World Radiocommunication Conference

3.4 Mathematical formulas

For the purposes of the present document, the following mathematical formulas will be used:

- $a \quad b$ means "a" equal "b" by definition
- $a = b$ means "a" equal "b" by a process of deduction, e.g.: $x \quad -3 \quad \Rightarrow \quad x^2 = 9$
- $a := b$ means "a" takes the value of "b", e.g.: $x := x+2 \quad \Rightarrow \quad \mathit{the\,value\,of\,x\,is\,incremented\,by}\,2$

- for any three vectors $\vec{a}, \vec{b}\, and\, \vec{c}$:

$$\begin{cases} \left(\vec{a} \wedge \vec{b}\right) = -\left(\vec{b} \wedge \vec{a}\right) \\ \left(\vec{a} \wedge \vec{b}\right).\vec{c} = \vec{a}.\left(\vec{b} \wedge \vec{c}\right) \\ \left(\vec{a} \wedge \vec{b}\right).\vec{c} = \vec{c}.\left(\vec{a} \wedge \vec{b}\right) \end{cases} \qquad \begin{cases} \left(\left(\vec{a} \wedge \vec{b}\right).\vec{b}\right).\vec{b} = -\left\|\vec{b}\right\|^2.\left(\vec{a} \wedge \vec{b}\right) \\ \vec{a} = \dfrac{1}{\left\|\vec{b}\right\|^2}.\left[\left(\vec{a}.\vec{b}\right).\vec{b} + \left(\vec{a} \wedge \vec{b}\right) \wedge \vec{b}\right] \end{cases} \quad (1)$$

$$\Upsilon(x) = \begin{cases} 0 & for\; x < 0 \\ 1 & for\; x \geq 0 \end{cases}, \text{ the Heaviside function,} \quad (2)$$

$$Sign(x) = \begin{cases} -1 & for\; x < 0 \\ 0 & for\; x = 0 \\ +1 & for\; x > 0 \end{cases}, \text{ the "sign of " function,} \quad (3)$$

$$ArcTan2(x, y) = \begin{cases} = \varphi & rad \quad if\; a \geq 0 \\ = \varphi + \pi & rad \quad if\; a < 0 \end{cases} \qquad for: \begin{cases} x = a.\sin(\varphi) \\ y = a.\cos(\varphi) \end{cases} \quad (4)$$

4 GSO arc protection requirement

Several ITU-R Recommendations and Resolutions specify off-axis EIRP density limits for the protection of the satellites within 3° of the GSO arc as it could be seen within the following examples:

- The ITU-R Recommendation S.728-1 [1] applicable to Ku band Very Small Aperture Terminals (VSATs) recommends "that VSAT earth stations operating with geostationary satellites in the 14 GHz frequency band used by the FSS be designed in such a manner that at any angle φ specified below, off the main-lobe axis of an earth-station antenna, the maximum EIRP in any direction within 3° of the GSO should not exceed" the specified values and for φ ≥ 2°.

- The ITU-R Recommendation S.580-6 [2] recommends design objectives for antenna radiation diagrams of earth stations operating with geostationary satellites which should be met for any off-axis direction which is within 3° of the GSO and for which: $\varphi_{min} \leq \varphi \leq 20°$ where φ_{min} is a function of the antenna diameter and the wave length and for an greater off-axis angles, φ, that the ITU-R Recommendation S.465-5 [3] should be used.

- ITU-R Recommendation S.524-8 [4] (Maximum permissible levels of off-axis EIRP density from earth stations in geostationary-satellite orbit networks operating in the fixed-satellite service transmitting in the 6 GHz, 14 GHz and 30 GHz frequency bands) recommends "that GSO networks in the FSS operating in the 6 GHz frequency band be designed in such a manner that at any angle, φ, which is 2,5° or more off the main lobe axis of an earth station antenna, the EIRP density in any direction within 3° of the GSO should not exceed" the specified limit. In Ka band the minimum value of φ is 2°. Additionally for Ku and Ka bands, "for any direction in the region outside 3° of the GSO, the specified limits may be exceeded by no more than 3 dB".

- The ITU-R Resolution 902 (WRC-03) [5] applicable to Earth Stations located on board Vessels (ESVs) requires that: "at any angle φ, etc., off the main-lobe axis of an earth-station antenna, the maximum EIRP in any direction within 3° of the GSO shall not exceed" the specified limits for φ ≥ 2,5° in C band and for φ ≥ 2° in Ku band.

and:

- The ITU-R Recommendation S.580-6 [2] in Note recommends 3 that: "When elliptical beam antennas are used the side-lobe radiation in the direction of the GSO can be reduced if the minor axis of the beam (major axis of the antenna) is oriented so that it is parallel to the GSO". Further study is required on the application of this Recommendation in the case of the minor axis of the antenna which would correspond with a D / λ < 50.

The present document is intended to show how to compute which parts of the antenna radiation pattern are affected by any of these requirements - antenna radiation patterns or the e.i.r.p. density - in order to provide protection of the satellites within the vicinity of the GSO.

5 Aspects of the GSO arc

5.1 Aspect of the GSO arc from the space

From the space the GSO is a circle of radius ρ = 42 164 km around the Earth and the GSO vicinity is the band within 3° of the GSO on the sphere of radius ρ around the Earth.

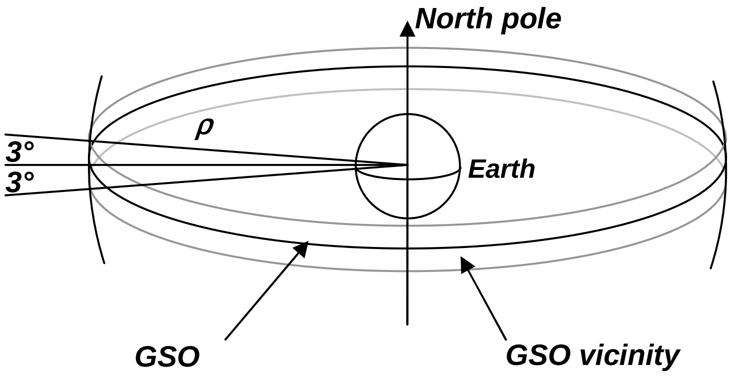

Figure 1: Aspect of the GSO arc from the space

5.2 Aspect of the GSO arc from an Earth Station (ES)

Only one portion of the GSO arc above the horizon is visible from an earth station. The GSO arc and the limits of the GSO vicinity look like ellipses.

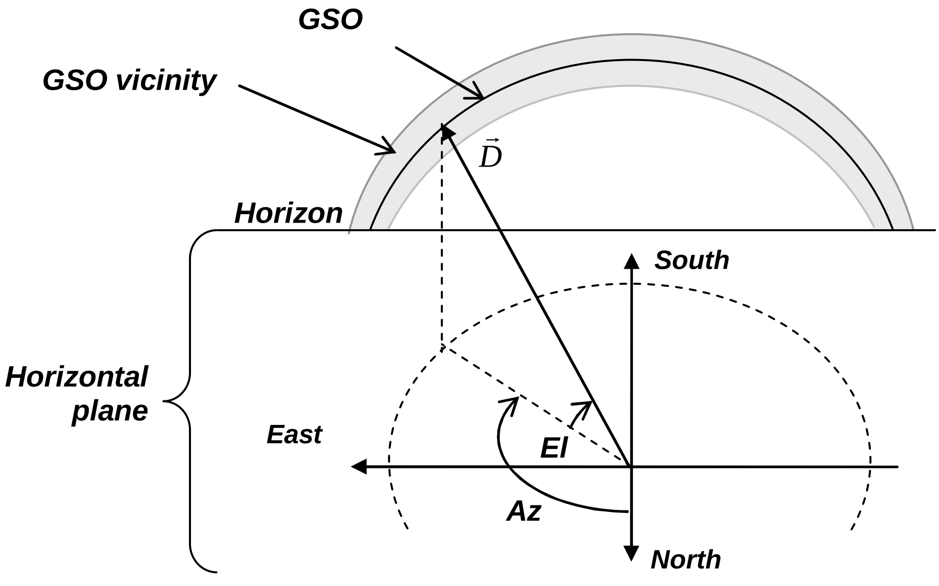

Figure 2: Aspect of the GSO arc from an earth station

Locally at the surface of the Earth any direction $\vec{D}$ may be defined by:

- the elevation angle (El) of the direction with the horizontal plane; and
- the azimuth angle (Az) of the projection of that direction on the horizontal plane with the local North direction.

On figure 3, the GSO arc and its vicinity are represented as a function of the elevation (El) and azimuth (Az) for different values of the earth station latitude (Lt).

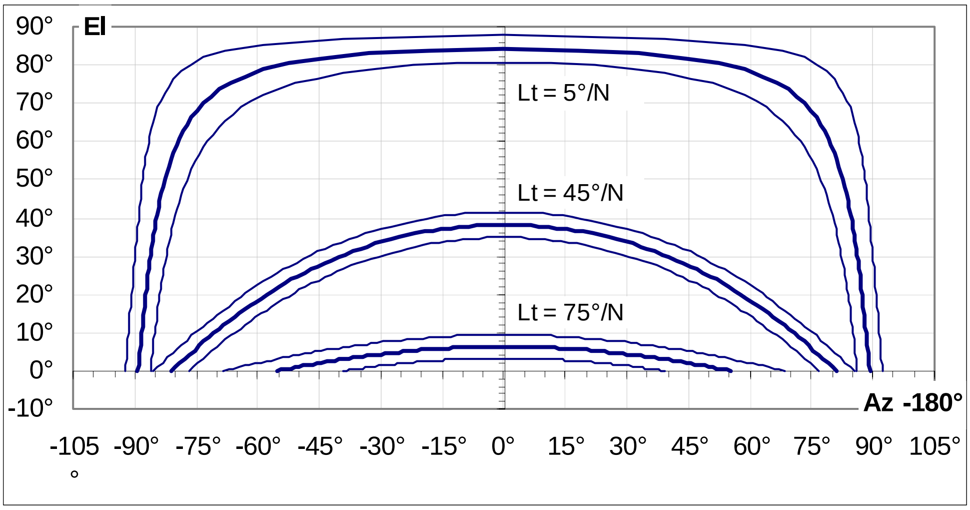

Figure 3: Azimuth (Az) and elevation (El) of the GSO arc and its vicinity for different values of the earth station latitude (Lt)

5.3 Aspect of the GSO arc from an ES antenna

An ES antenna may be represented by its reflector or the -3 dB contour of its 3-dimension (3-D) antenna radiation pattern.

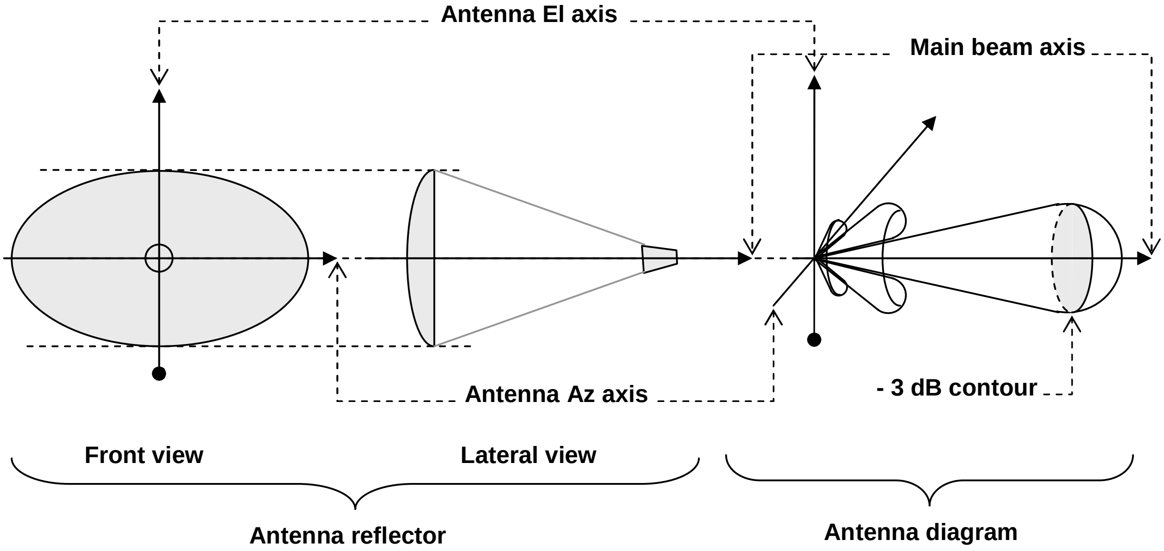

Figure 4: Antenna reflector and 3-D antenna radiation pattern

Three axes, as represented on figures 4 and 5, are defined on an antenna:

- the antenna main beam axis: the direction where the antenna gain is maximum;
- the antenna azimuth axis (Az-axis): the direction towards the left hand side and parallel to the horizontal plane when the antenna is oriented toward the South;
- the antenna elevation axis (El axis): the direction parallel to the local vertical when the antenna is oriented toward the South with its main beam axis horizontal.

The directions of these 3 axis are dependent on the antenna orientation as shown on figure 5.

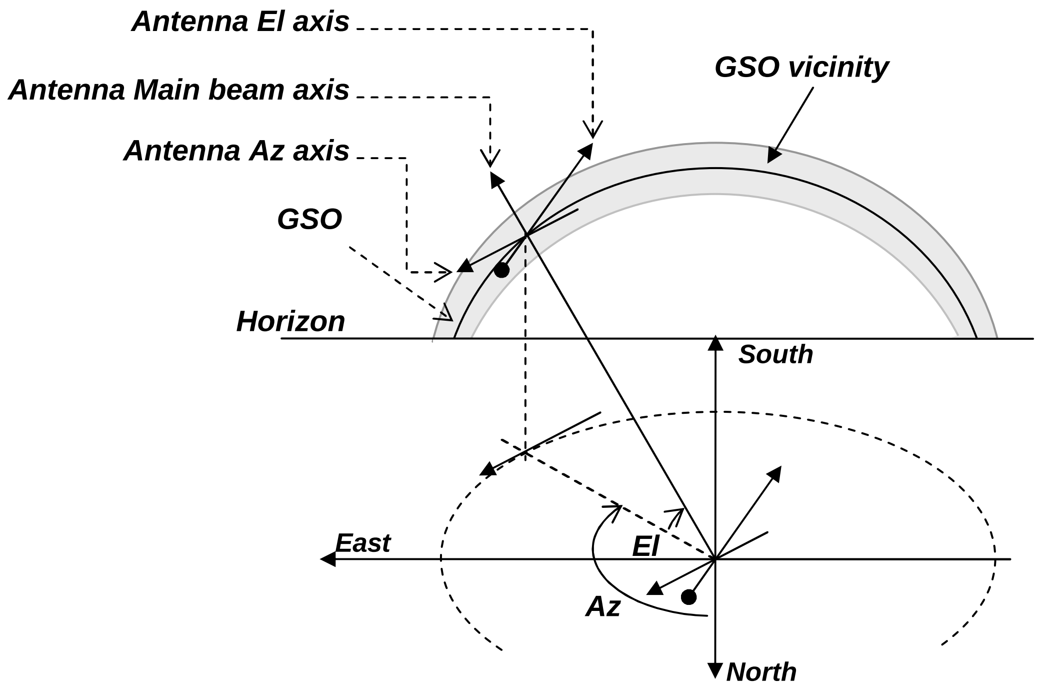

Figure 5: Antenna axes and GSO arc

Any direction $\vec{D}$ from the antenna may be defined by a couple (φ_{Az}, φ_{El}) of angles, represented on figure 6, and where:

- the angle φ_{Az} is the angle of the projection of the considered direction on the plane orthogonal to the antenna elevation axis (El axis) with the antenna main beam axis (Az-axis); and
- the angle φ_{El} is the angle of the considered direction with the plane orthogonal to the antenna elevation axis (El axis).

When the satellite is within the meridian plane of the antenna, then for any direction in the vicinity of the satellite:

- the angle φ_{Az} is approximately equal to the azimuth angle of the considered direction minus 180°; and
- the angle φ_{El} is approximately equal to the elevation angle of the considered direction minus the elevation of the satellite direction.

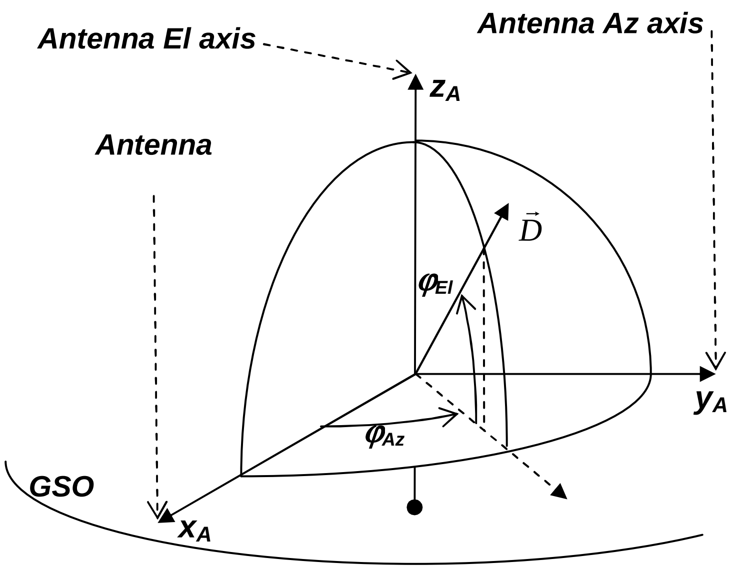

Figure 6: Antenna axes and angles φ_{AZ} and φ_{EL} of a direction $\vec{D}$

NOTE: The coordinate system used within the present document (see figure 6) may be different from other antenna coordinate systems. In particular, the present document uses the x-axis as the antenna main beam axis instead of the z-axis.

On figure 7, the GSO arc and its vicinity are represented as a function of the angles φ_{AZ} and φ_{EL} for an earth station at 0°/E and 45°/N and for different values of the satellite longitude (Lg_S).

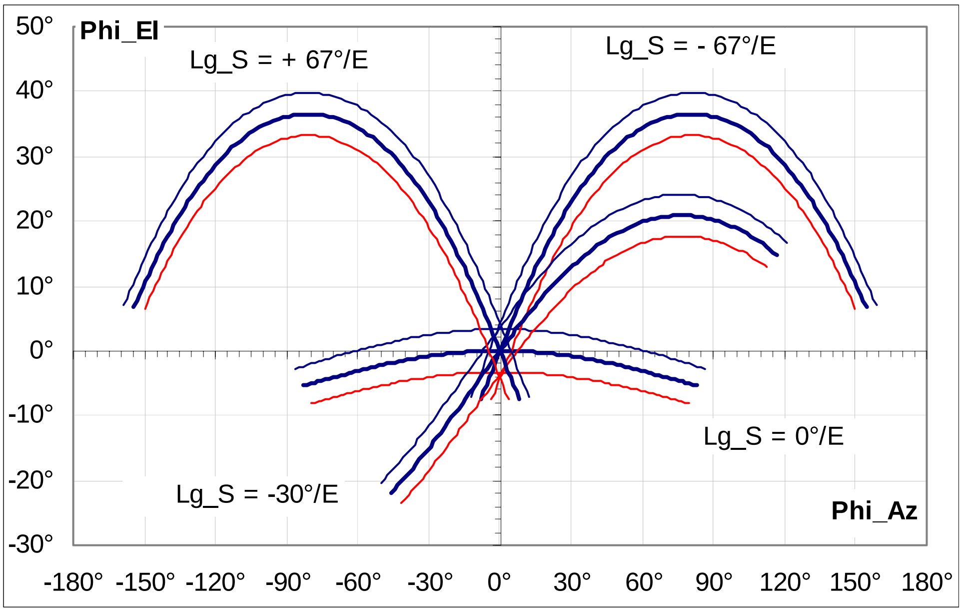

Figure 7: GSO arc and its vicinity for an earth station at 0°/E and 45°/N and pointed towards different satellites at longitude Lg_S

Any direction $\vec{D}$ from the antenna may also be defined by a couple (α, φ) of angles, represented on figure 8, where:

- the angle α is the angle of the projection of the considered direction on the plane orthogonal to the antenna main beam direction with the antenna azimuth axis (Az-axis); and
- the angle φ is the off-axis angle of the considered direction with the antenna main beam direction.

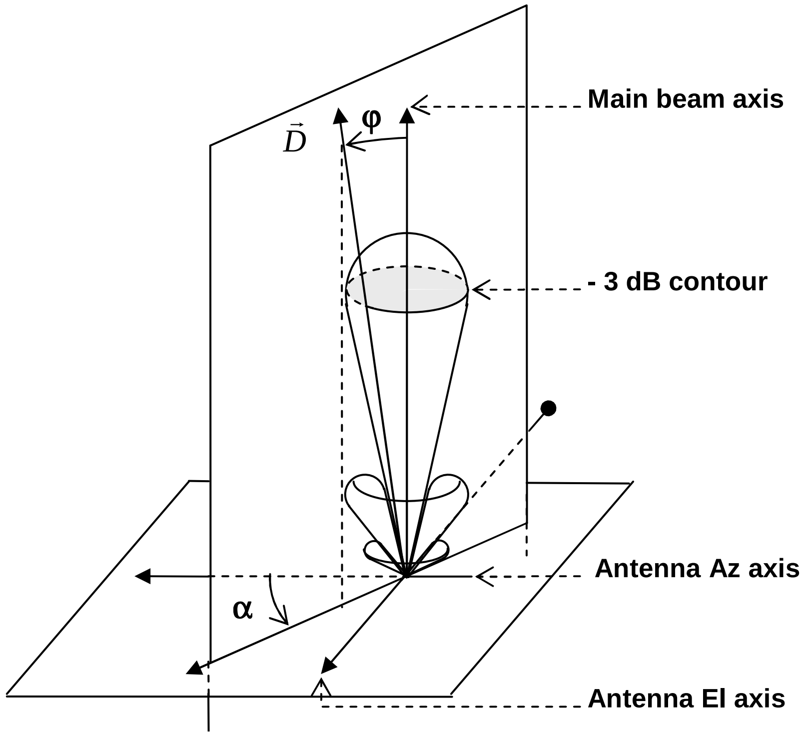

Figure 8: Off-axis angles (φ), and α angle of a direction $\vec{D}$

The antenna radiation pattern is usually measured within 2 or more planes containing the main bean axis:

- within the plane α = 0, containing the antenna Az-axis;
- within the plane α = 90°, containing the antenna El axis; and
- when necessary, e.g. with non symmetrical antennas, within intermediate planes (e.g. for α = - 45° and α = +45°).

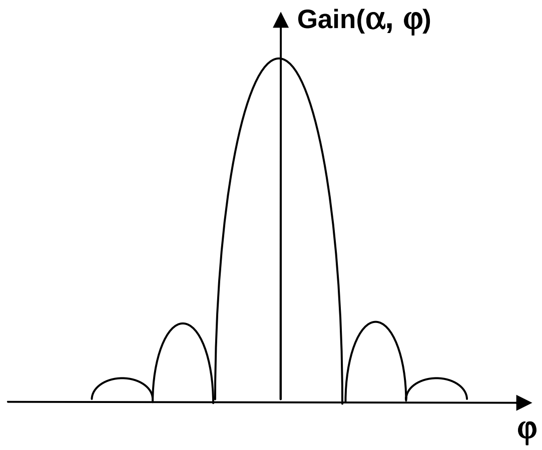

Figure 9: Antenna radiation pattern measured in a given plan (α = constant)

The GSO arc and its vicinity will be represented within the (α, φ) domain in order to determine which parts of the antenna radiation patterns have to meet the requirements for the protection of the GSO arc.

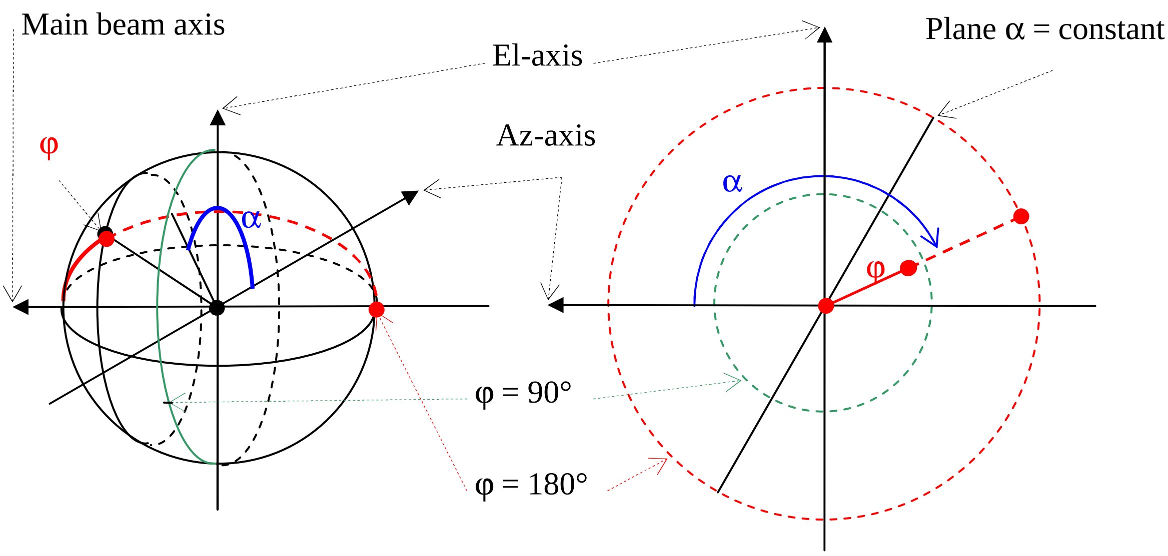

Figure 10: (α, φ) domain for the representation of planes within the antenna radiation pattern

For an antenna which always has its azimuth axis parallel to the horizontal plane, the GSO arc and its vicinity may cover a large range within the off-axis angle domain (φ, α).

5.3.1 Types of antenna mounts

Three types of antenna mounts are considered:

- the azimuth-elevation antenna mounts without possibility of alignment of the antenna Az-axis with the GSO tangent;
- the azimuth-elevation antenna mounts with alignment of the antenna Az-axis with the GSO tangent, with two sub-cases:
  - the theoretical case;
  - the case where the polarizer is not rotating but fixed, the two polarization planes rotate with the antenna and one of these two planes is aligned with the electric field received from the satellite;
- the equatorial antenna mounts.

More precisely, what is called "alignment of the antenna Az-axis with the GSO tangent" consists in:

- putting into coincidence the plane defined by the antenna main beam axis and the antenna Az-axis with the plane defined by the antenna main beam axis and the GSO tangent; or
- putting orthogonal the El-axis with the GSO tangent as shown on figure 11.

For an observer located behind the antenna and looking at the satellite in the direction of the antenna main beam axis, the GSO tangent and the antenna Az-axis appear aligned, even though there are not parallel.

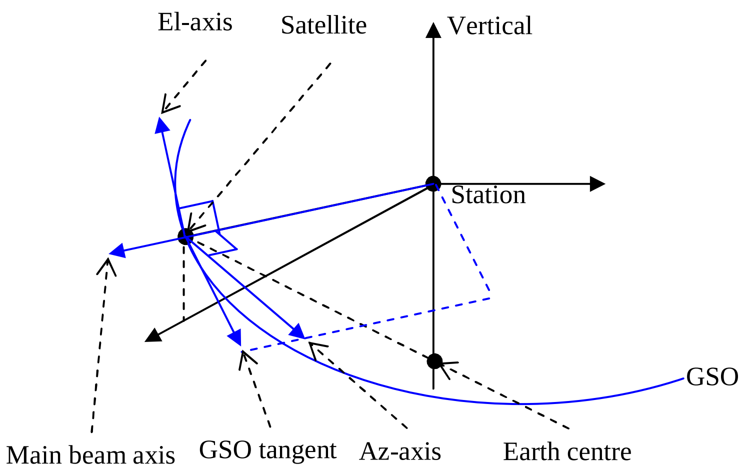

Figure 11: Alignment of the antenna Az-axis with the GSO tangent

5.3.2 Azimuth-elevation antenna mount without GSO tangent alignment

An azimuth-elevation antenna mount without GSO tangent alignment capability consists in two axes:

- the antenna mount azimuth axis which is vertical;
- the antenna mount elevation axis which is horizontal.

In the case of an azimuth-elevation antenna mount without GSO tangent alignment, the antenna orientation relative to the antenna mount elevation axis is constant for any direction of the antenna main beam axis.

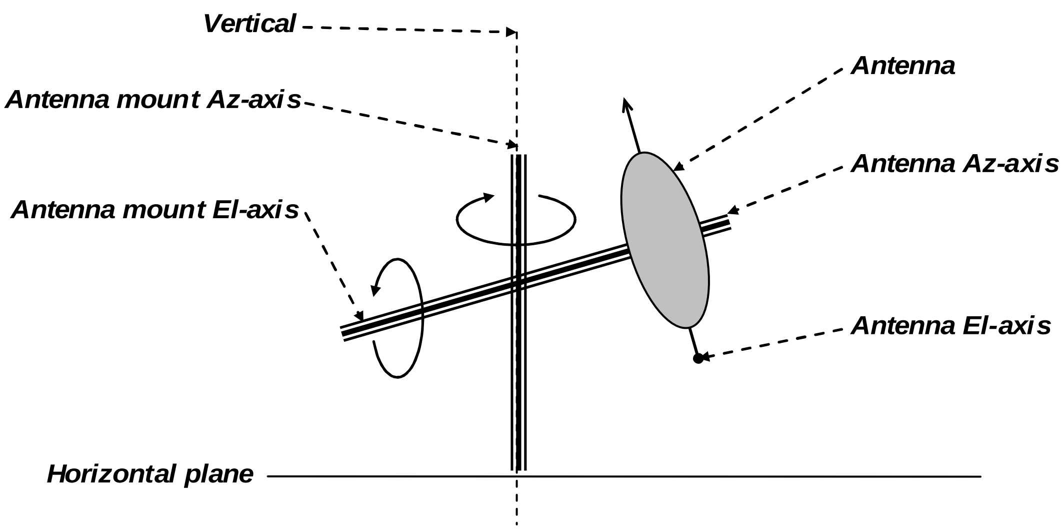

Figure 12: Azimuth-elevation antenna mount with no GSO tangent alignment

The antenna Az-axis is always parallel to the horizontal plane at the earth station. Figure 13 represents the visible part of GSO arc and its vicinity, the -3 dB contour of the antenna main beam and the relative position of the antenna azimuth and elevation axes for various satellite positions on the GSO arc.

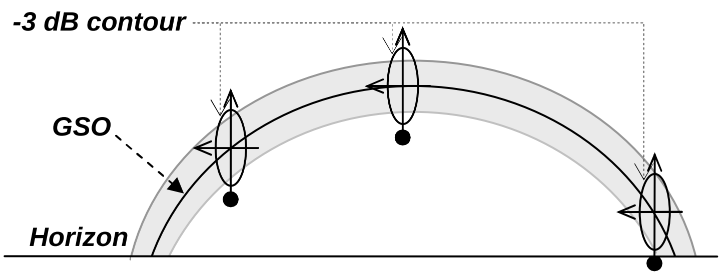

Figure 13: GSO arc and -3 dB contour of the antenna main beam for various satellite positions

Figure 14 represents the GSO arc and its vicinity within the antenna (α, φ) domain for various ES latitudes and satellite positions on the GSO arc.

Case	ES latitude	Lg_S0
(1)	5,000°/N	74,313°/E
(2)	30,000°/N	71,880°/E
(3)	70,000°/N	38,046°/E
(4)	30,000°/N	-71,880°/E
(5)	1,000°/N	0,744°/E

Case (5) corresponds to the case of an ES in the vicinity (e.g. at 138 km) of the sub-satellite point on the Earth.

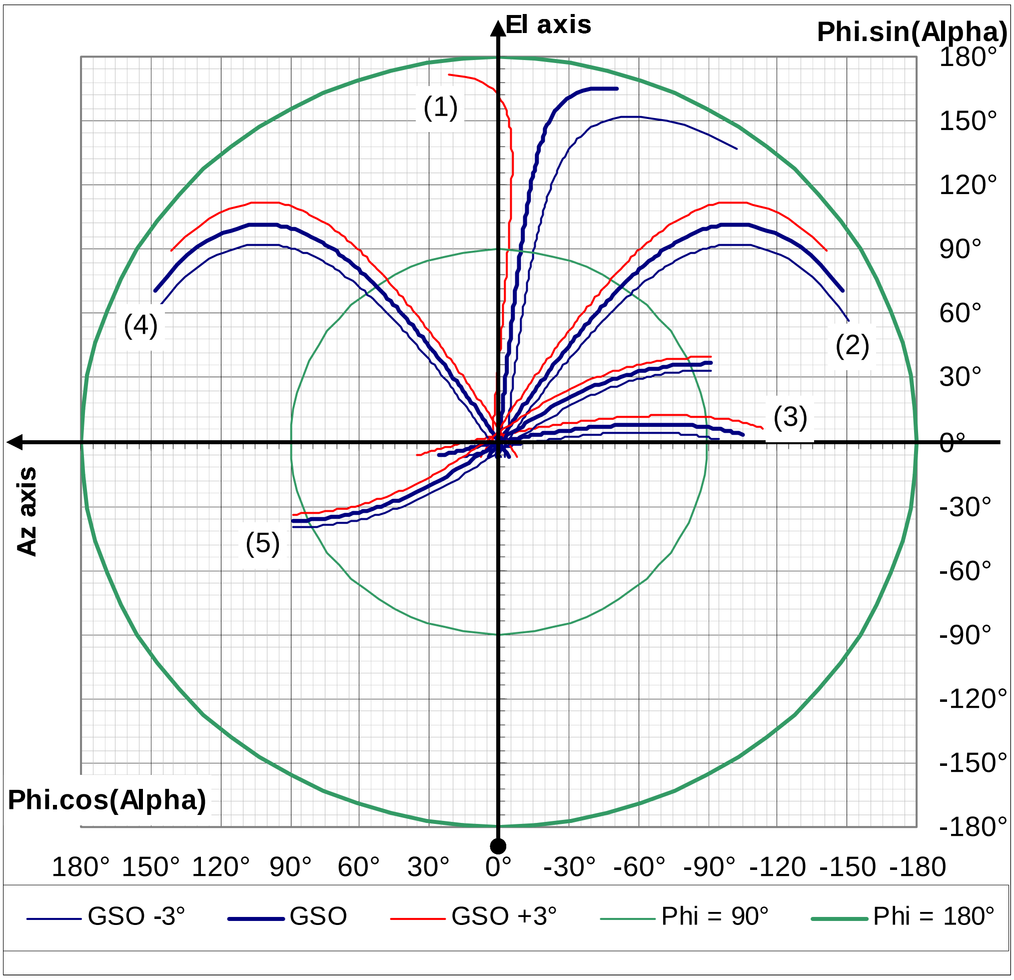

Figure 14: GSO arc and its vicinity within the (α, φ) domain

For a given range of ES latitudes, a given minimum elevation angle of the pointed satellite, a given minimum elevation of the satellites to be protected, the envelope of projections or shadows of the GSO arc and its vicinity on the (α, φ) domain for all the possible positions of the pointed satellite, can be determined. It will be called the (α, φ) domain of the shadow of the GSO arc and its vicinity on the antenna radiation pattern. It is represented on figure 15 for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from -74°/N to 74°/N.

The part of contour which is approximately a circle of radius equal to 90°, corresponds to an ES located clause to the sub-satellite point on the Earth.

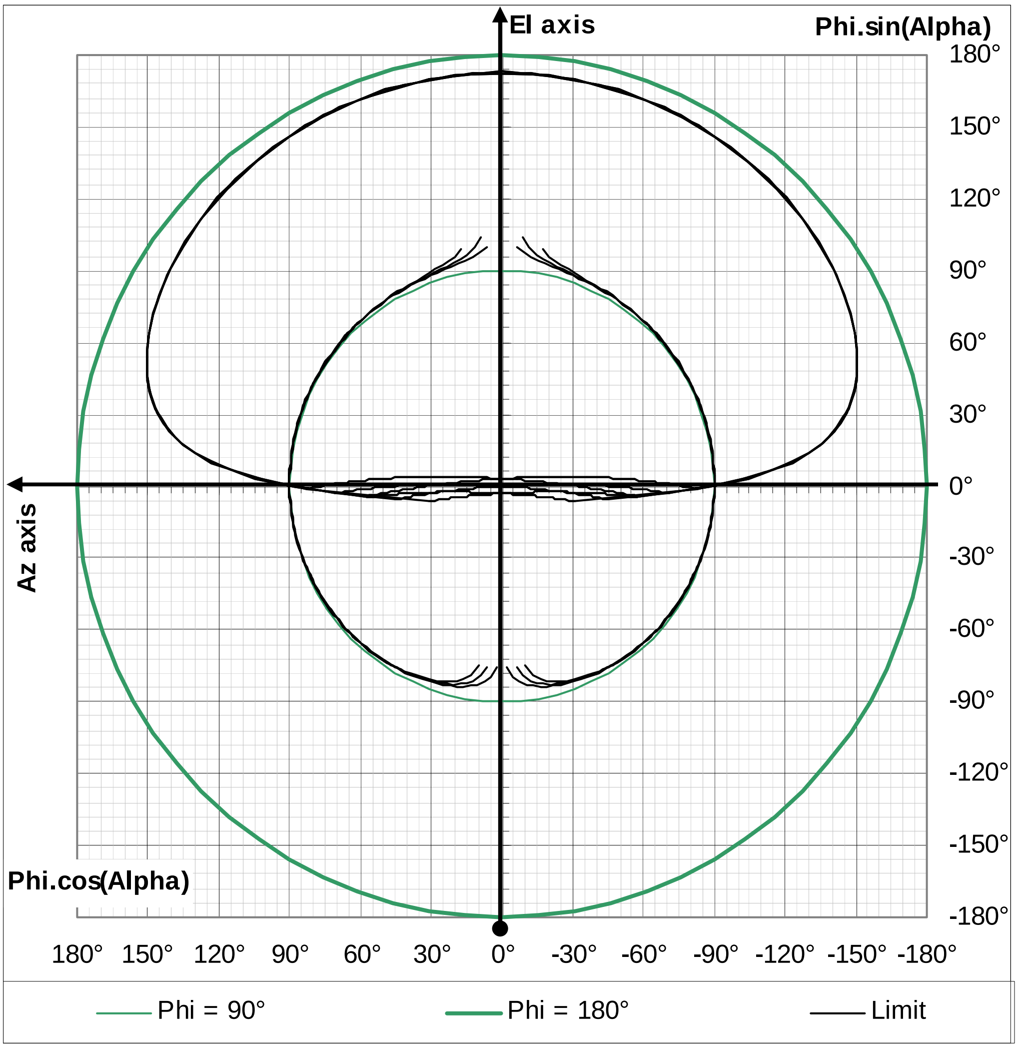

Figure 15: Limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from -74°/N to 74°/N

Due to the complexity of the contour, the contour has been computed for three adjacent satellite latitudes (-3°, 0° and +3°), and for different cases:

- the cases where the ES latitude varies from the minimum latitude to the maximum latitude, the ES antenna is successively pointed towards the western and eastern satellites at the minimum elevation, and the satellites to be protected are successively the eastern and western satellites at the minimum elevation;
- the cases where the ES latitude is maximum, the ES antenna is successively pointed towards the western and eastern satellites at the minimum elevation, and the longitude of the satellite to be protected varies from the eastern to the western satellite longitudes at the minimum elevation. If the absolute value of the ES maximum latitude is lower than 1° then computations are made with 1° instead of the ES maximum latitude;
- the cases where the ES latitude is minimum, the ES antenna is successively pointed towards the western and eastern satellites at the minimum elevation , and the longitude of the satellite to be protected varies from the eastern to the western satellite longitudes at the minimum elevation. If the absolute value of the ES minimum latitude is lower than 1° then computations are made with 1° instead of the ES minimum latitude;
- the cases where the ES latitude is minimum, the longitude of the satellite pointed by the ES antenna varies from the eastern to the western satellite longitudes at the minimum elevation, and the longitude of the satellite to be protected is successively the eastern to the western satellite longitudes at the minimum elevation. If the absolute value of the ES minimum latitude is lower than 1° then computations are made with 1° instead of the ES minimum latitude.

This method of computation the contour has also been used for the other type of antenna mount.

In the following clause it will be shown that for an antenna which has always its azimuth axis aligned with the tangent to the GSO arc, the GSO arc and its vicinity covers a smaller range within the off-axis angle domain (φ, α).

For an antenna designed for a limited range of latitudes, the GSO arc and its vicinity covers a smaller range within the off-axis angle domain (φ, α), as shown on figure 16 for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from 35°/N to 65°/N.

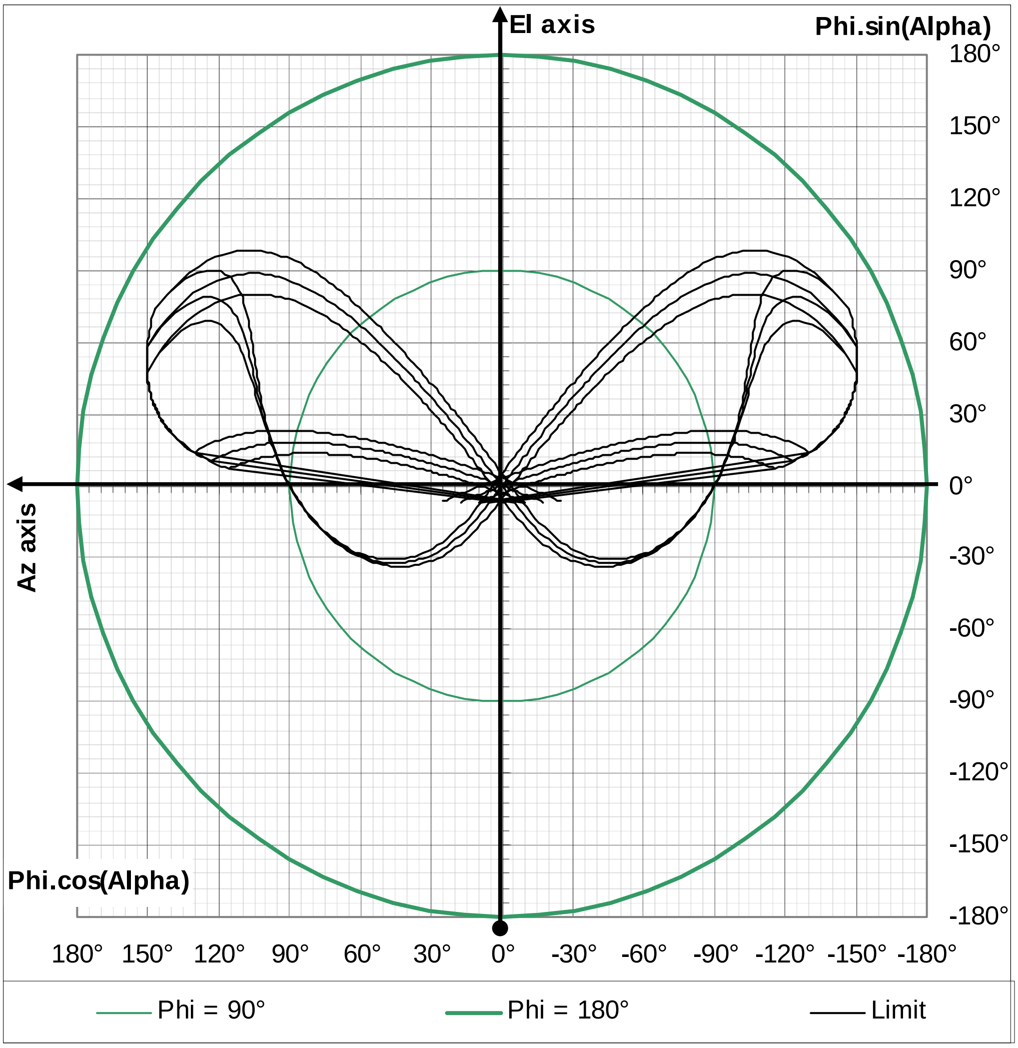

Figure 16: Limit within the (α, φ) domain of the projection of the GSO arc and its vicinity for latitudes from 35°/N to 65°/N

With an alignment error or offset of the antenna Az-axis the contour is rotated as shown on figure 17 for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from 0°/N to 74°/N;
- an offset of the antenna Az-axis of +5°;

and on figure 18 for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from -74°/N to 74°/N;
- an offset of the antenna Az-axis of +5°.

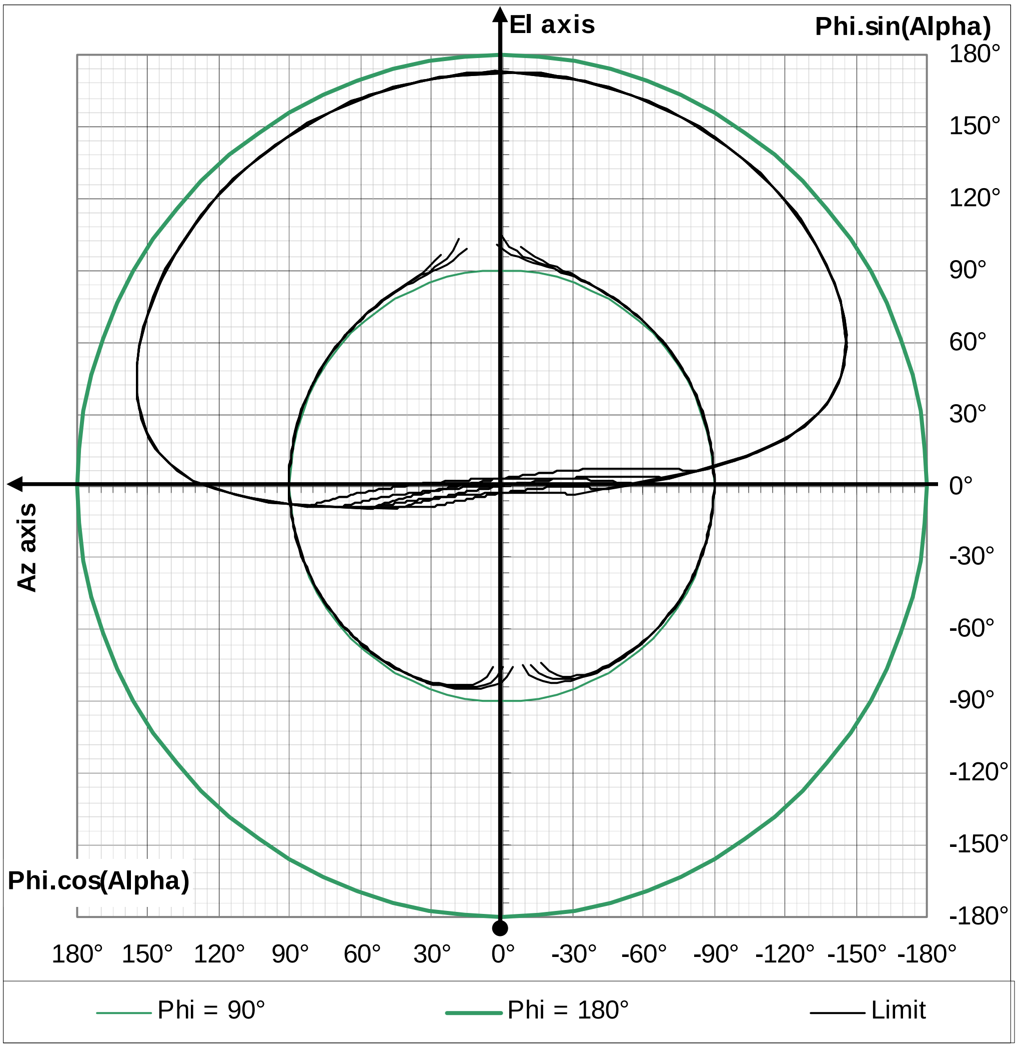

Figure 17: Limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from -74°/N to 74°/N and with an alignment offset of +5°

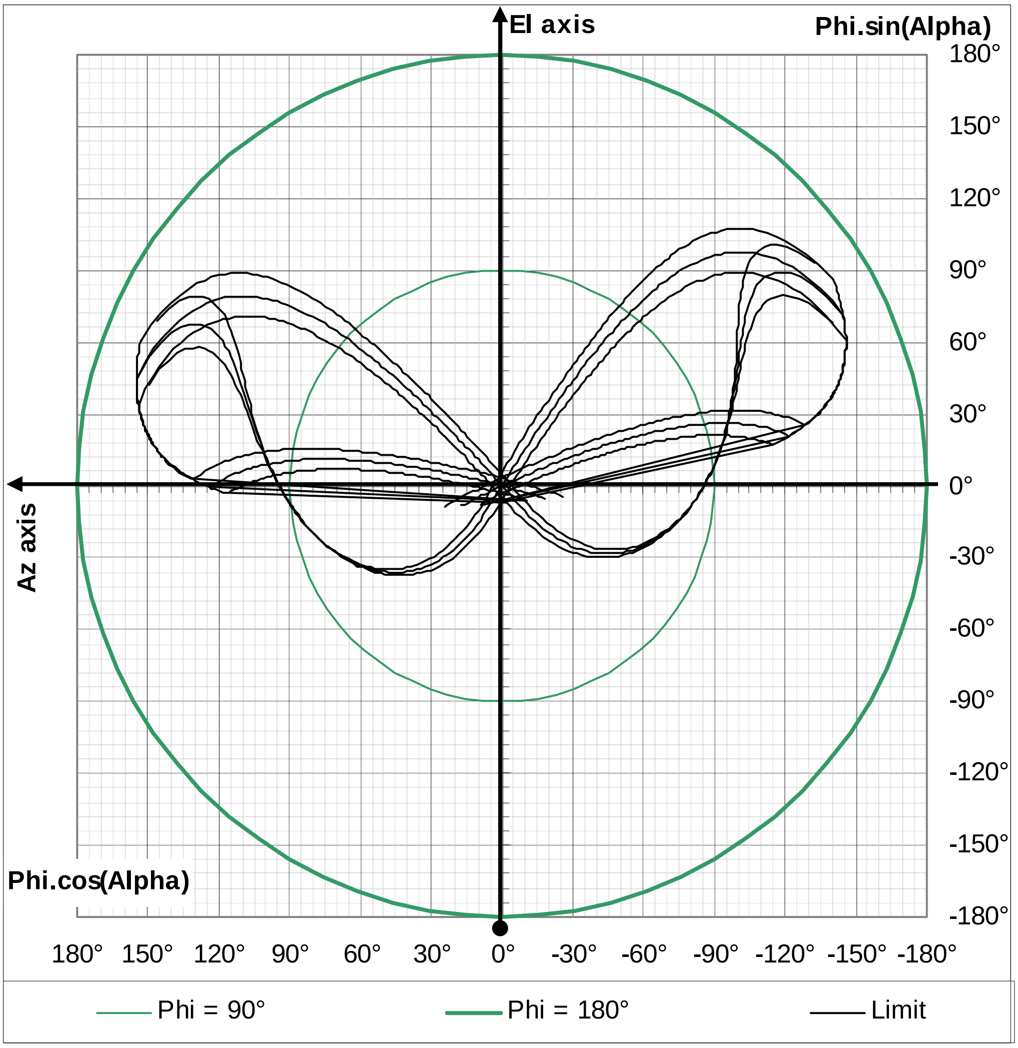

Figure 18: Limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from 35°/N to 65°/N and with an alignment error of 5°

It is obvious that a permanent offset i of the Az-axis result in a rotation of i the contour. This is verified on figures 17 and 18.

For antennas which could be used up-side down, the contour has to be computed for offset $i = 0°$ and for $i = 180°$.

It will be demonstrated that:

- For a antenna with an azimuth-elevation mount without GSO tangent alignment, designed for a maximum elevation $El_{S,\max}$ and for a maximum vertical axis offset $\hat{\theta}_{\max}$ then the maximum value $\delta i_{\max}$ of the additional offset δi of the antenna azimuth axis $\overrightarrow{u_{Az,S}}$, due to misalignment, is given by the following equations:

- for $0 \le \left|El_{S,\max}\right| \le \frac{\pi}{2} - \left|\hat{\theta}_{\max}\right|$: $$\delta i_{\max} = ArcSin\left(\frac{\sin\left(\left|\hat{\theta}_{\max}\right|\right)}{\cos\left(\left|El_{S,\max}\right|\right)}\right) \quad (4a)$$

- for $\frac{\pi}{2} - \left|\hat{\theta}_{\max}\right| \le \left|El_{S,\max}\right| \le \frac{\pi}{2}$: $$\delta i_{\max} = \frac{\pi}{2} rad \quad (4b)$$

This value δi_{max} is indirectly a function of the ES latitude and has to be added, for one extreme case, and subtracted, for the other extreme case, to the permanent offset i for the computation of the GSO shadow on the antenna radiation pattern, for each ES latitude.

5.3.3 Azimuth-elevation antenna mount with GSO tangent alignment

5.3.3.1 General

An azimuth-elevation antenna mount with GSO tangent alignment capability consists in three axes:

- the antenna mount azimuth axis which is vertical;
- the antenna mount elevation axis which is horizontal;
- the antenna mount "attitude" axis which is parallel to the antenna main beam axis.

In the case of an azimuth-elevation antenna mount with GSO tangent alignment capability, the antenna orientation relative to the antenna mount elevation axis is adjustable for any direction of the antenna main beam axis.

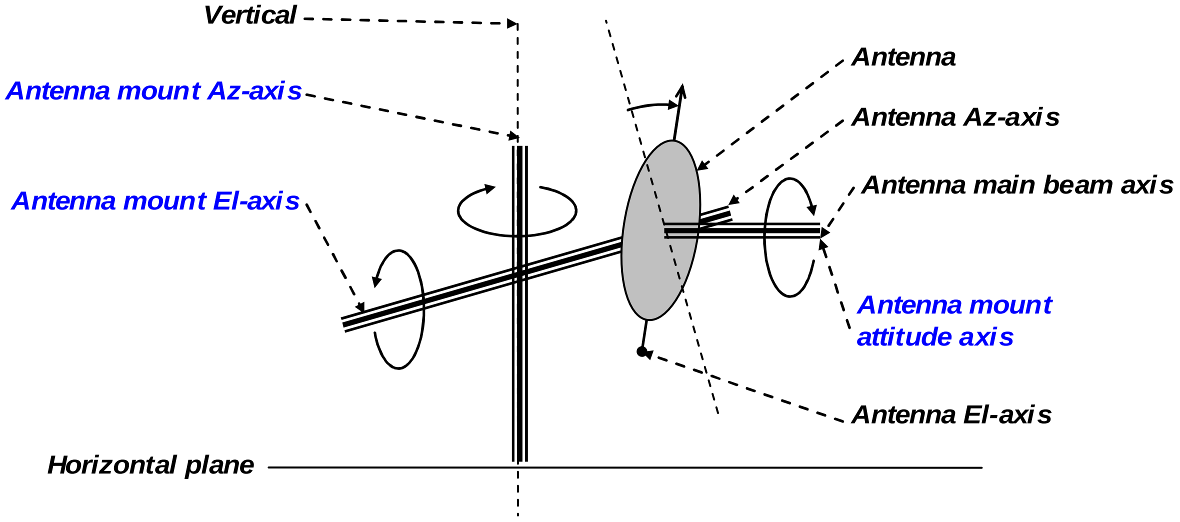

Figure 19: Azimuth-elevation antenna mount with GSO tangent alignment

The antenna El-axis is set orthogonal to the GSO tangent. Figure 20 represents the visible part of GSO arc and its vicinity, the -3 dB contour of the antenna main beam and the relative position of the antenna azimuth and elevation axes for various satellite positions on the GSO arc.

In fact there is no practical means of knowing that such antenna is perfectly aligned with the GSO arc. This case is presented as the ideal case which gives the smallest (α, φ) domain of the GSO arc shadow on the antenna radiation pattern.

A practical case close to that ideal case consists in making the antenna polarizer fixed but on the antenna instead of being rotating as usual.

The ideal case is presented first, and then case of an antenna with a fixed polarizer is presented.

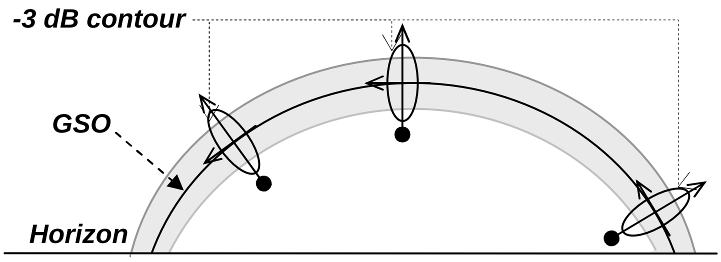

Figure 20: GSO arc and -3 dB contour of the antenna main beam for various satellite positions

5.3.3.2 Ideal case

Figure 21 represents the GSO arc and its vicinity within the antenna (α, φ) domain for various ES latitudes and satellite positions on the GSO arc.

Case	ES latitude	Lg_S0
(1)	5,000°/N	74,313°/E
(2)	30,000°/N	71,880°/E
(3)	70,000°/N	38,046°/E
(4)	30,000°/N	-71,880°/E
(5)	1,000°/N	0,744°/E

Case (5) corresponds to the case of an ES in the vicinity (e.g. at 138 km) of the sub-satellite point on the Earth.

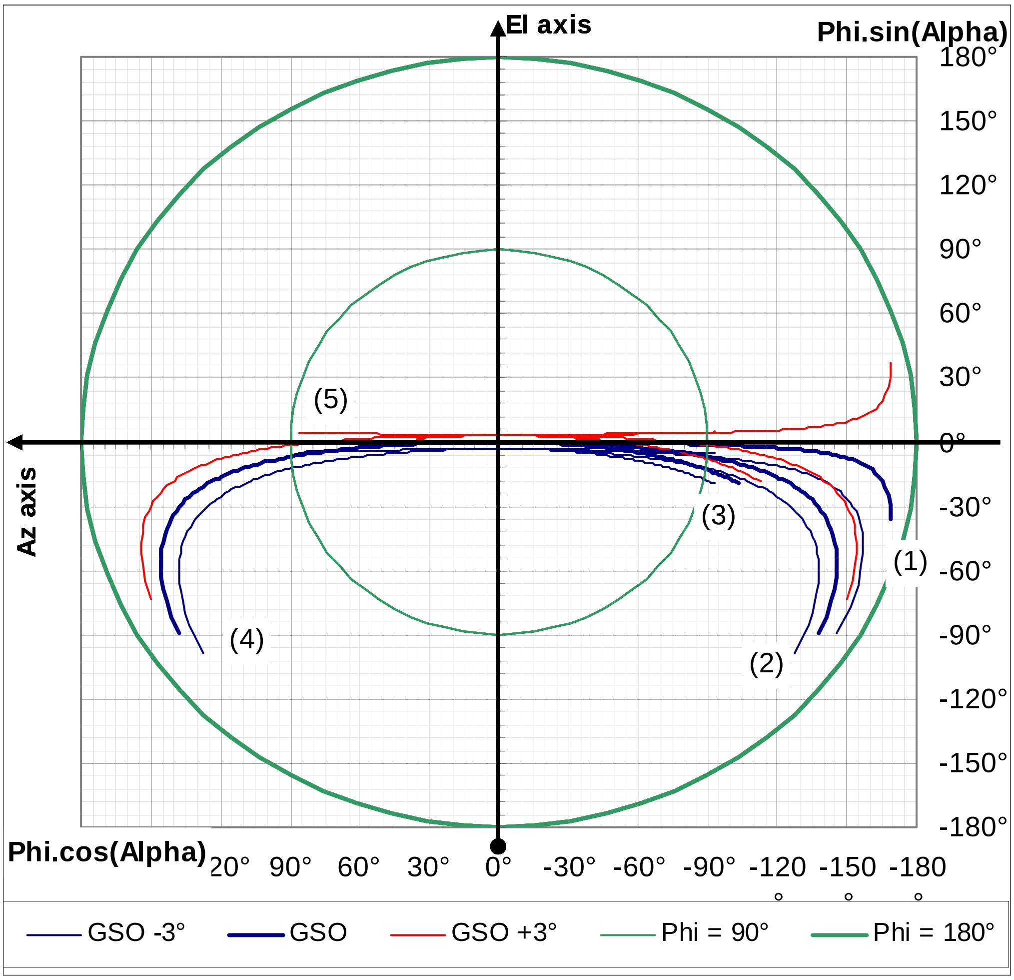

Figure 21: GSO arc and its vicinity within the (α, φ) domain

The (α, φ) domain of the shadow of the GSO arc and its vicinity on the antenna radiation pattern is represented on figure 22 for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from -74°/N to 74°/N.

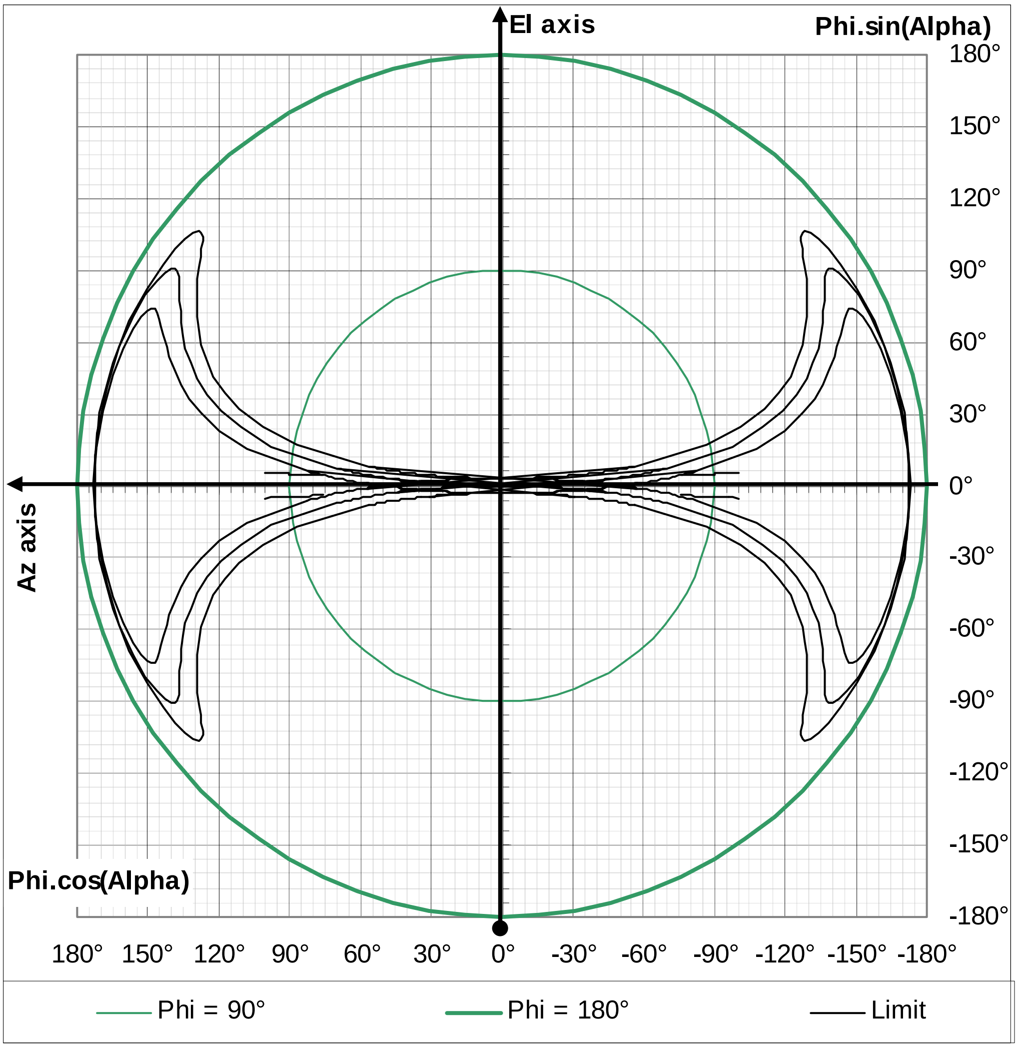

Figure 22: Limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from -74°/N to 74°/N

The above contour has been computed for 3 different latitudes of the adjacent satellite: -3°, 0° and +3°, using the method described for the Az-El antenna mount without GSO alignment.

With an alignment error or offset of the antenna Az-axis the contour is rotated as shown on figure 23 for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from 0°/N to 74°/N;
- an offset of the antenna Az-axis of +5°.

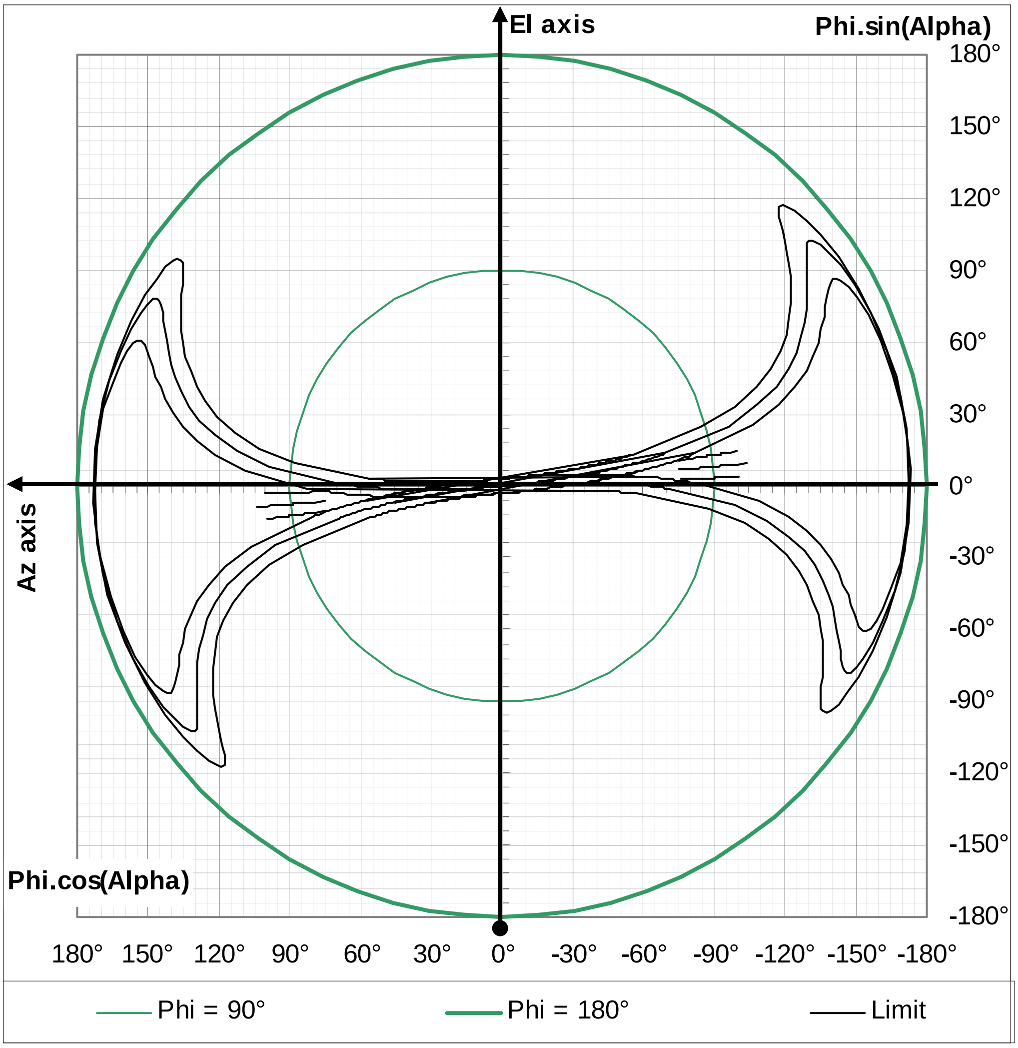

Figure 23: Limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from -74°/N to 74°/N and with an alignment error of 5°

It is obvious that a permanent offset i of the Az-axis result in a rotation of i the contour. This is verified on figure 23.

For antennas which could be used up-side down, the contour has to be computed for offset $i = 0°$ and for $i = 180°$.

It will be demonstrated that:

- For a antenna with an azimuth-elevation mount with GSO tangent alignment the value of the additional offset δi of the antenna azimuth axis $\overrightarrow{u_{Az,S}}$ is always equal to zero:

$$\boxed{\delta i = 0}$$

  what ever are the values of the elevation and of the vertical axis offset.

5.3.3.3 Practical case of an antenna with a fixed polarizer

In that case the polarizer is not rotating but fixed, the two polarization planes rotate with the antenna and one of these two planes is aligned with the electric field received from the satellite.

In order to estimate the (α, φ) domain of the shadow of the GSO arc and its vicinity on the antenna radiation pattern the satellite antenna is assumed to radiate an electric field $\overrightarrow{E_0}$ and the associated magnetic field $\overrightarrow{H_0}$ in the direction of the centre S_c of the antenna beam coverage so that the direction of the received electric field $\overrightarrow{E_{r,n}}$ and the associated magnetic field $\overrightarrow{H_{r,n}}$ at station S_n may be computed. The electric field $\overrightarrow{E_0}$ may have a tilt angle (i_E0) with the pole direction.

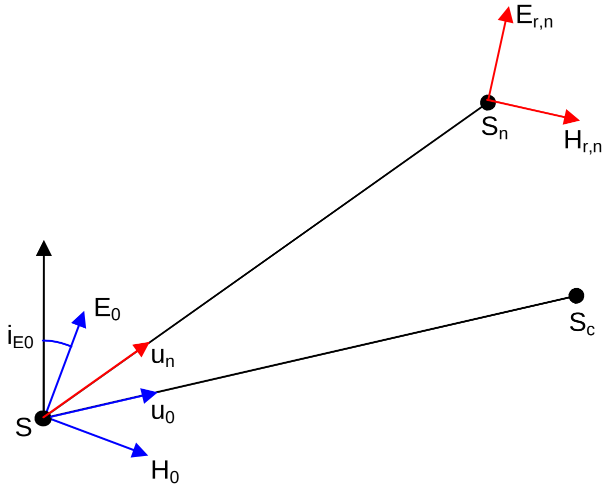

Figure 24: E and H fields at the satellite and at the ES

The (α, φ) domain of the shadow of the GSO arc and its vicinity on the antenna radiation pattern is represented on figure 25 for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from -74°/N to 74°/N;
- a tilt angle (i_E0) with the North pole direction of the electric field at the satellite of 0°;
- a longitude of the centre (Lg_c) of the satellite beam coverage of 5°/E;
- a latitude of the centre (Lt_c) of the satellite beam coverage of 50°/N;
- with alignment on the received H field or the E field but with an additional rotation of 90°.

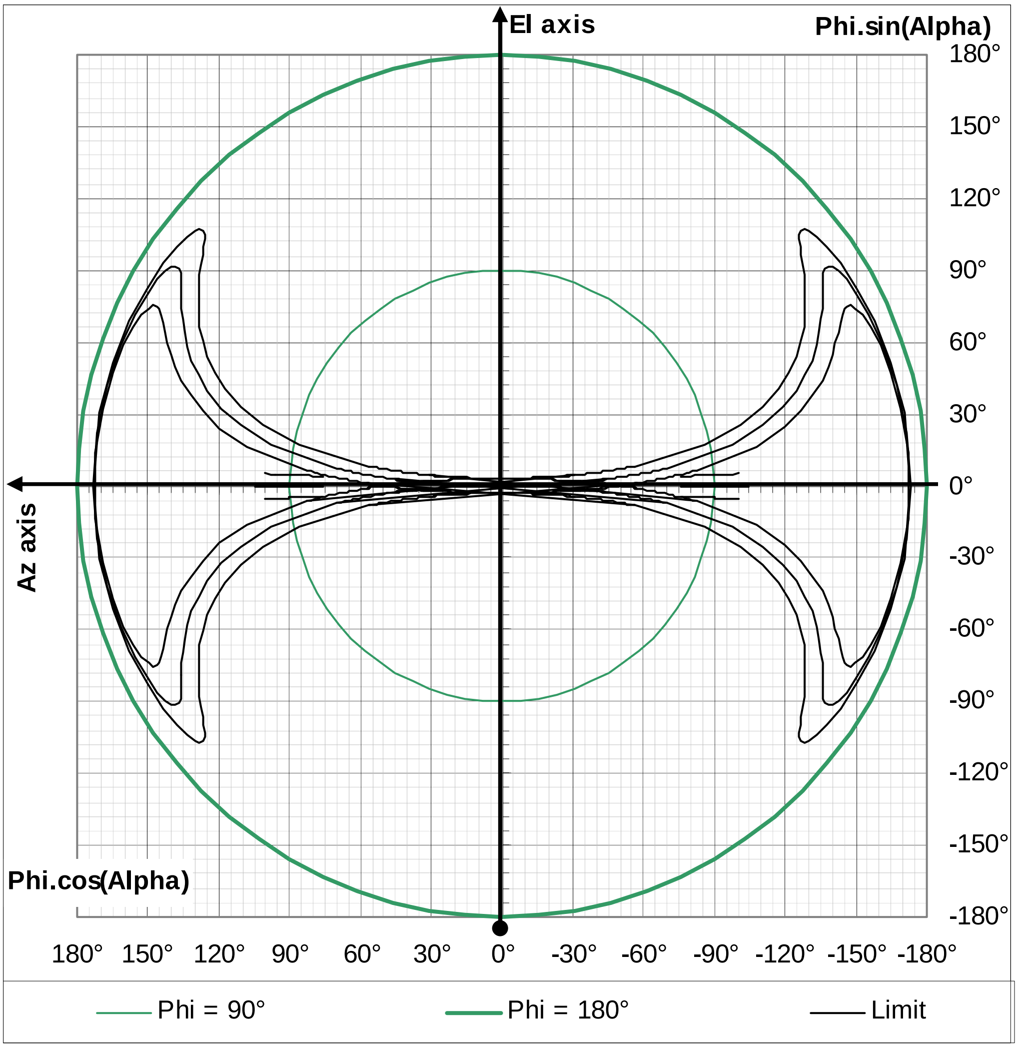

Figure 25: Limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from -74°/N to 74°/N and Lg_c = 5°, Lt_c = 50°, i_E0 = 0°

With an alignment error or offset of the antenna Az-axis the contour is rotated as shown on figure 26 for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from -74°/N to 74°/N;
- a tilt angle (i_E0) with the North pole direction of the electric field at the satellite of 0°;
- a longitude of the centre (Lg_c) of the satellite beam coverage of 5°/E;
- a latitude of the centre (Lt_c) of the satellite beam coverage of 50°/N;
- an offset of the antenna Az-axis of +5°;
- with alignment on the received H field or the E field but with an additional rotation of 90°.

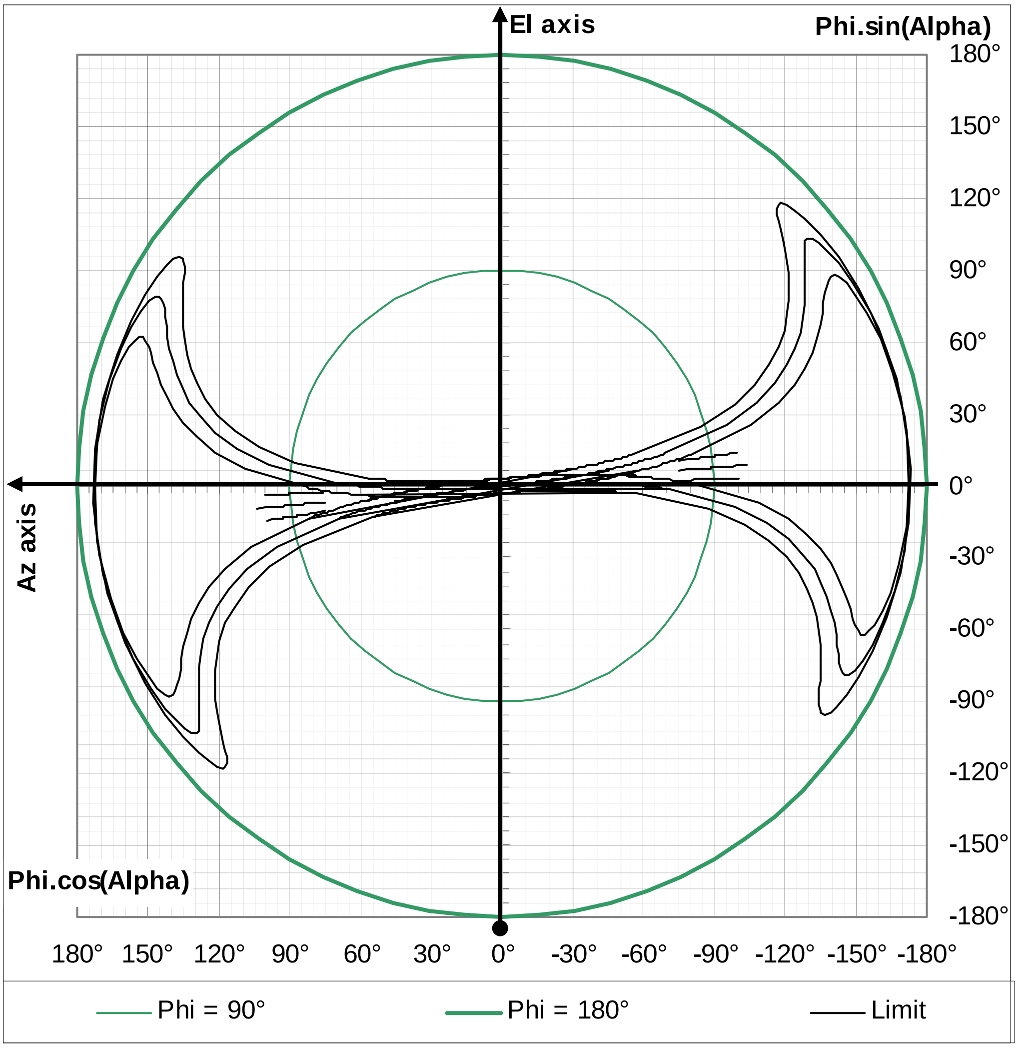

Figure 26: Limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from -74°/N to 74°/N, Lg_c = 5°, Lt_c = 50°, i_E0 = 0° and with an alignment error of 5°

It is obvious that a permanent offset i of the Az-axis result in a rotation of i the contour. This is verified on figure 27.

For antennas which could be used up-side down, the contour has to be computed for offset $i = 0°$ and for $i = 180°$.

With a tilt angle of the electric field radiated by the satellite equal to 22°, as for Telecom 2 satellites in Ku band, the contour is no more ideal as shown on figure 27 for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from -74°/N to 74°/N;
- a tilt angle (i_E0) with the North pole direction of the electric field at the satellite of 22°;
- a longitude of the centre (Lg_c) of the satellite beam coverage of 5°/E;
- a latitude of the centre (Lt_c) of the satellite beam coverage of 50°/N;
- no offset of the antenna Az-axis;
- with alignment on the received H field or the E field but with an additional rotation of 90°.

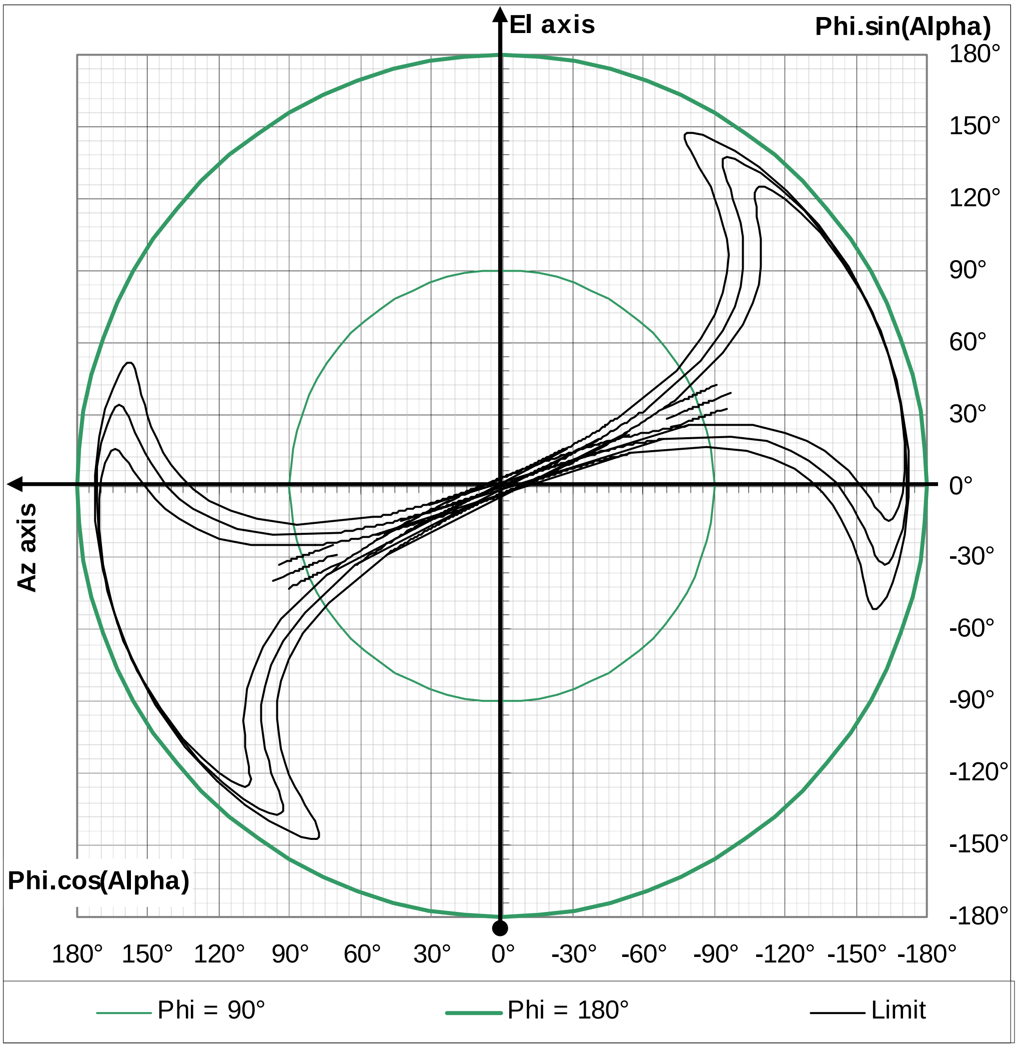

Figure 27: Limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from -74°/N to 74°/N, Lg_c = 5°, Lt_c = 50°, i_E0 = 22° and with no alignment error

When the centre of the satellite beam coverage is not at the sub-satellite point on the Earth, as in the above example, this implies that the coverage area is limited to a portion of the visible part of the Earth from the satellite, and consequently that the range of operational latitudes of the ES is also limited to a smaller range than the range from -74°/N to 74°/N, in the above example.

When the range of operational latitudes of such antenna is limited to the range from 30°/N to 74°/N, with a tilt angle of the electric field radiated by the satellite equal to 22°, as for Telecom 2 satellites in Ku band, the contour is still close to the ideal contour but it is rotated as shown on figure 28 for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from 30°/N to 74°/N;
- a tilt angle (i_E0) with the North pole direction of the electric field at the satellite of 22°;
- a longitude of the centre (Lg_c) of the satellite beam coverage of 5°/E;
- a latitude of the centre (Lt_c) of the satellite beam coverage of 50°/N;
- no offset of the antenna Az-axis;
- with alignment on the received H field or the E field but with an additional rotation of 90°.

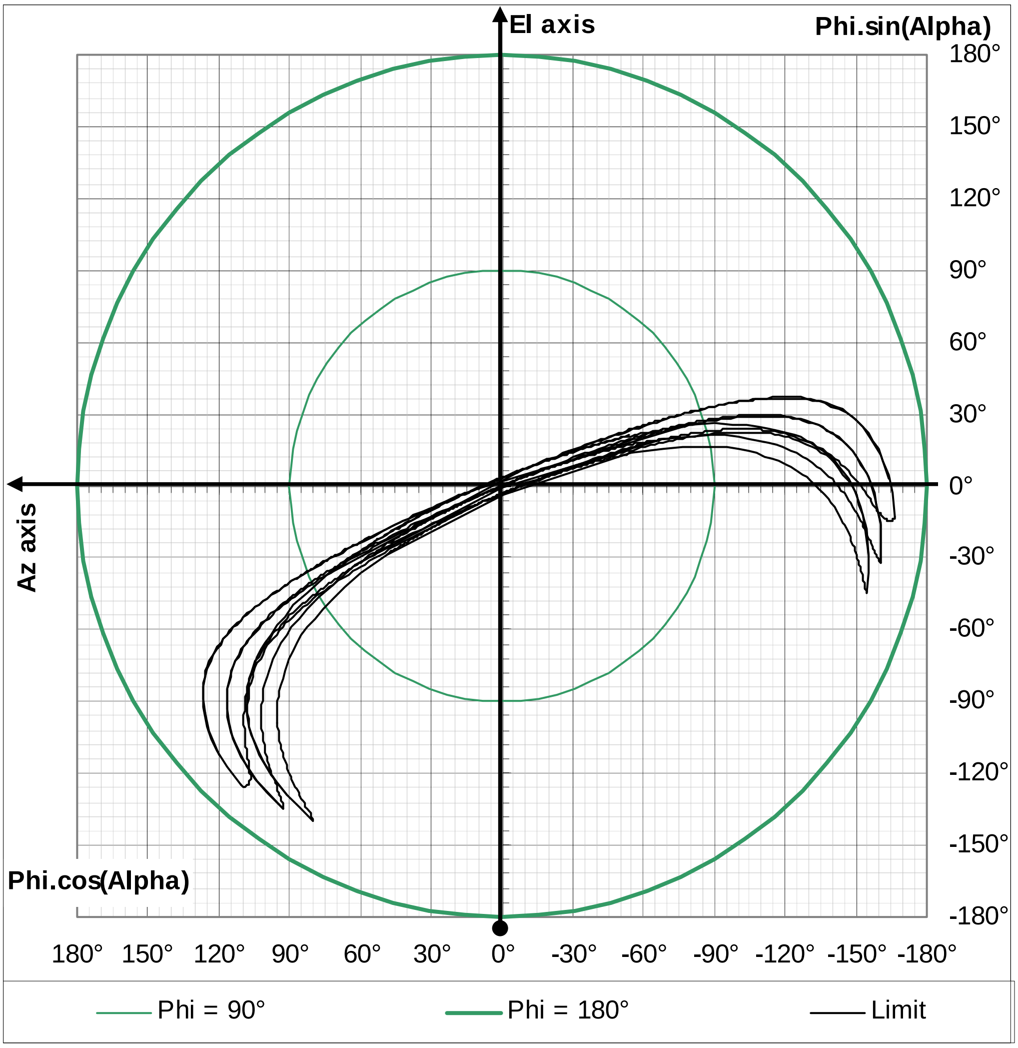

Figure 28: Limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from 30°/N to 74°/N, Lg_c = 5°, Lt_c = 50°, i_E0 = 22° and with no alignment error

Theses performances would be quite ideal if the antenna mount is fitted with a means of putting an offset equal to the electric field tilt angle as shown on figure 29 for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from 30°/N to 74°/N;
- a tilt angle (i_E0) with the North pole direction of the electric field at the satellite of 22°;
- a longitude of the centre (Lg_c) of the satellite beam coverage of 5°/E;
- a latitude of the centre (Lt_c) of the satellite beam coverage of 50°/N;
- an offset of the antenna Az-axis of -22°;
- with alignment on the received H field or the E field but with an additional rotation of 90°.

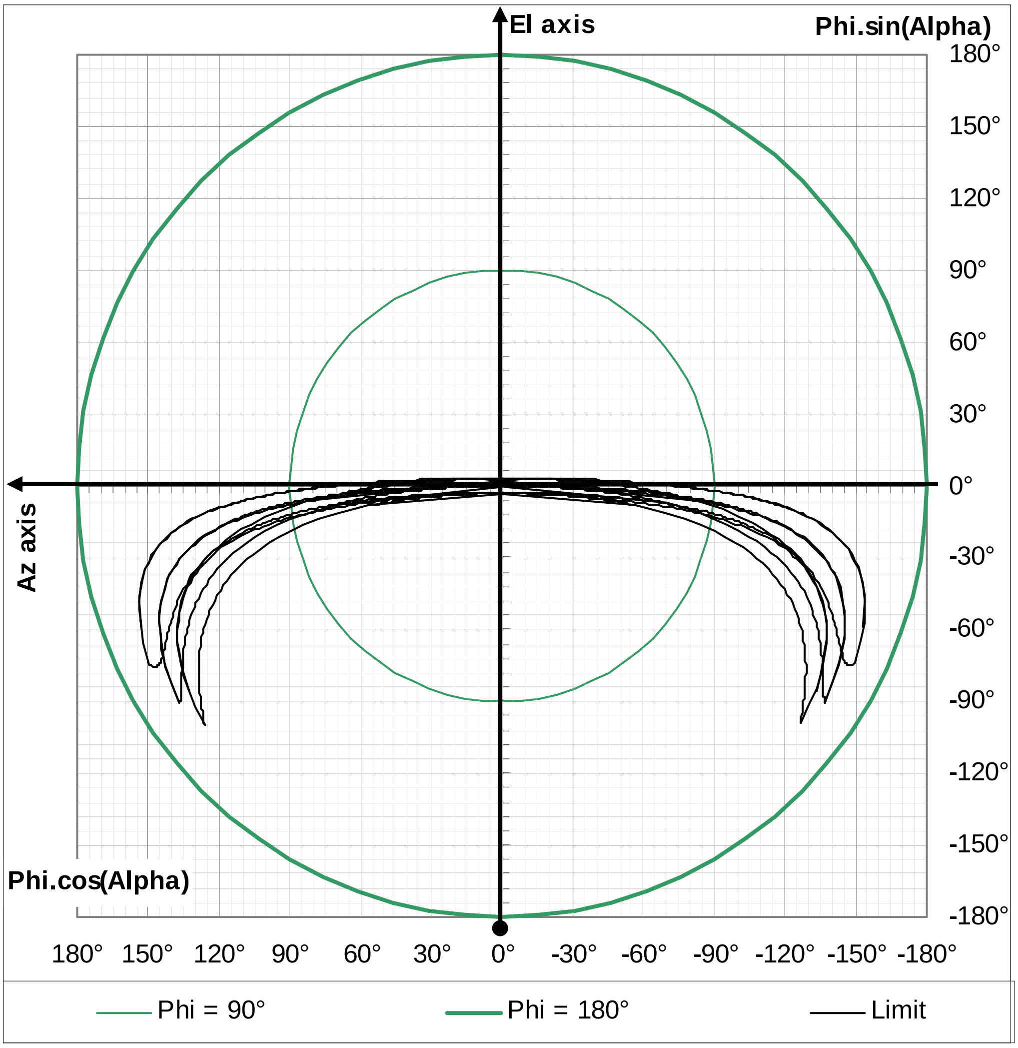

Figure 29: Limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from 30°/N to 74°/N, Lg_c = 5°, Lt_c = 50°, i_E0 = 22° and with an alignment offset of -22°

The tilt angle (i_E0) with the North Pole direction of the electric field at the satellite depends of the satellite and of the considered coverage and transmit frequency band. It is equal to 22° for Telecom 2 satellites. It seems that for some satellite operators the common values of the tilt angles is 0°, 3° or 7,5°, but it is not a general rule.

Presently, the values of the tilt angles seem to be not available in the satellite documentation published on the Web. Consequently the design of antenna using this technique of alignment with the GSO tangent is unsuitable for any satellite but could be suitable for a specific satellite or a series of satellites with the same value of the tilt angle, provided that the user is informed of that limitation, e.g. within the user documentation.

The value of the offset δi of the antenna azimuth axis $\overrightarrow{u_{Az,S}}$ is the sum of:

- the offset i_E0, if applied, for compensation of the tilt angle i_E0 of the electric field radiated by the satellite;
- the error δi_{E0} made in applying this offset;
- the error δi_{P_S} due to a difference of orientation of the electric field radiated in the direction of the ES with the electric field radiated in the direction of the centre of coverage;
- the error $\delta i_{P_{Sn}}$ due to a difference of orientation of the polarization plane of the receive antenna in the satellite direction but within the tracking or pointing contour of the antenna main beam with the polarization plane in the direction of the antenna main beam axis;

- the tracking or pointing alignment error $\delta i_{P\,alignment}$ made when aligning the antenna polarization plan with the received electric field;
- the error $\delta i_{ionosphere+rain}$ equal to the rotation angle of the received electric field through the ionosphere and through the atmosphere mainly when it is raining at the time of the alignment of the antenna polarization plan with the received electric field.

The error $\delta i_{ionosphere+rain}$ is important in C-band and not negligible in Ku band during rainy conditions. For this reason:

- the initial alignment of the polarization plane of antennas receiving in Ku band with the GSO tangent should be performed during clear sky conditions;
- the alignment of the polarization plane of antennas receiving in Ku band with the GSO tangent should not be performed during rainy sky conditions, consequently the automatic and permanent alignment systems are unsuitable for these antennas;
- the alignment of the polarization plane of antennas receiving in C band with the GSO tangent is unsuitable for antennas receiving in C band.

For the use of such antennas, investigations have to be performed on the range of values of the above listed errors and on their effects on the contour.

5.3.4 Equatorial antenna mount

An equatorial antenna mount normally consists of one axis but is in fact in three axes:

- the antenna mount pole axis which is parallel to the Earth pole axis;
- the antenna mount azimuth axis which is vertical;
- the antenna mount elevation axis which is horizontal.

The antenna mount azimuth axis and elevation axis are used to give the correct orientation to the antenna mount pole axis. Once done, the antenna only rotates around the antenna mount pole axis.

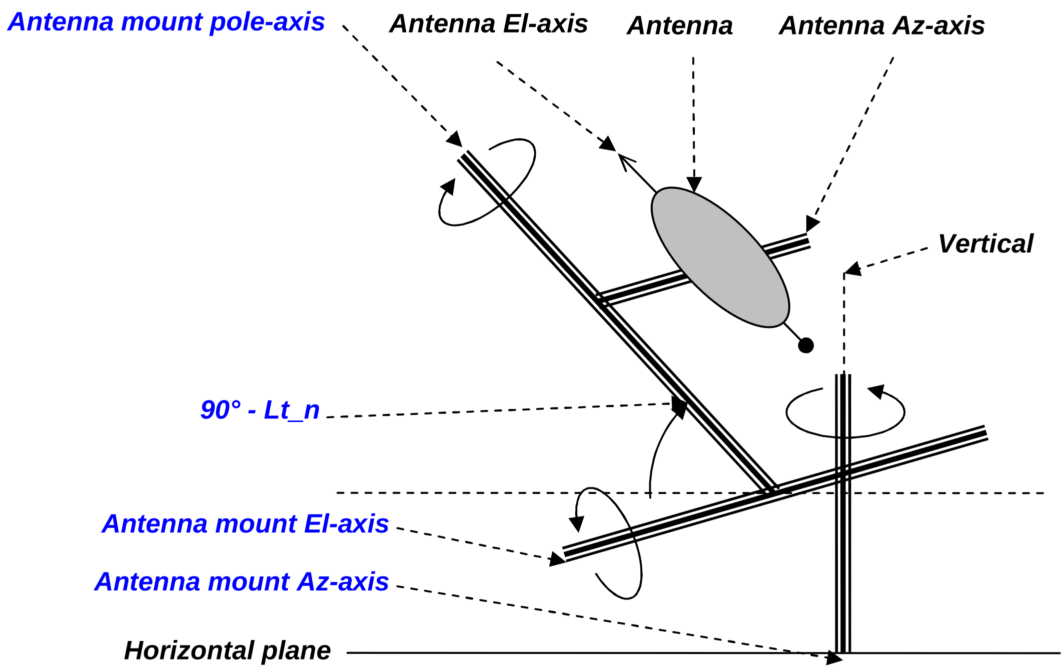

Figure 30: Equatorial antenna mount

Figure 31 represents the GSO arc and its vicinity within the antenna (α, φ) domain for various ES latitudes and satellite positions on the GSO arc.

Case	ES latitude	Lg_S0
(1)	5,000°/N	74,313°/E
(2)	30,000°/N	71,880°/E
(3)	70,000°/N	38,046°/E
(4)	30,000°/N	-71,880°/E
(5)	1,000°/N	0,744°/E

Case (5) corresponds to the case of an ES in the vicinity (e.g. at 138 km) of the sub-satellite point on the Earth.

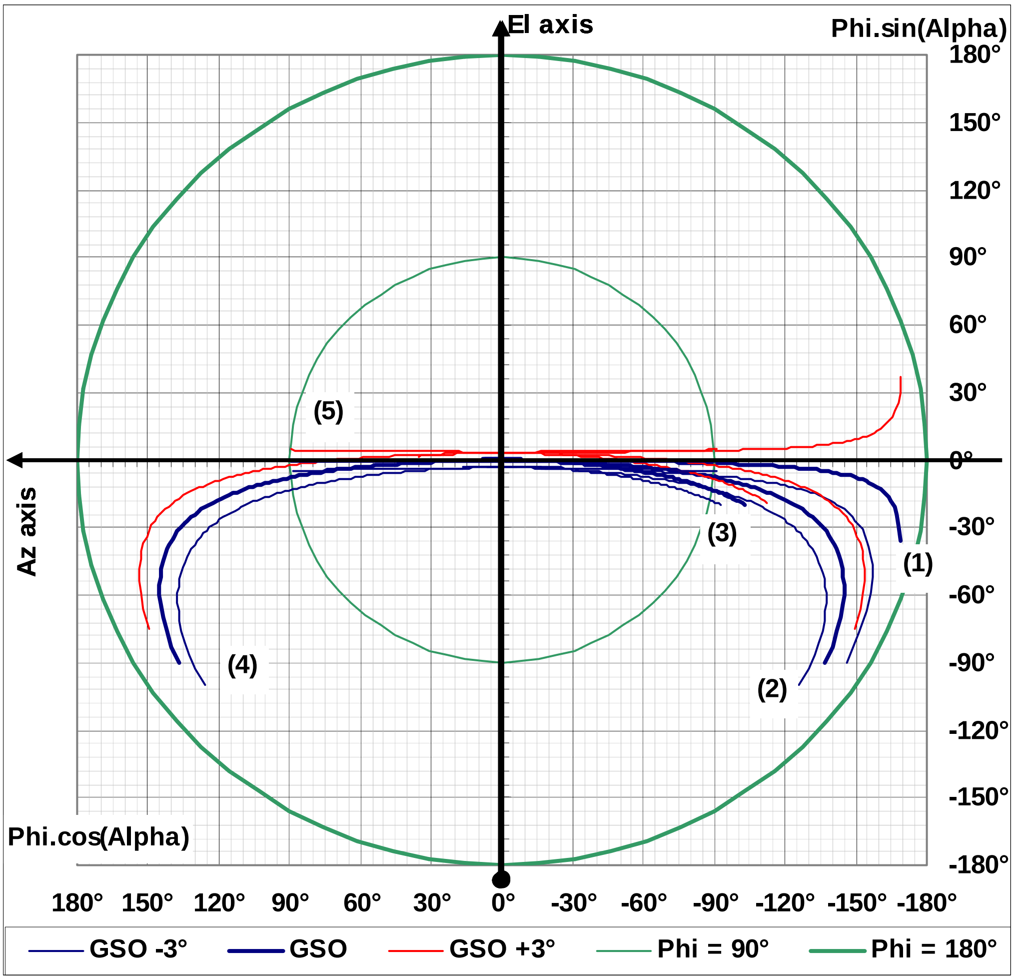

Figure 31: GSO arc and its vicinity within the (α, φ) domain

The (α, φ) domain of the shadow of the GSO arc and its vicinity on the antenna radiation pattern is represented on figure 32 for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from -74°/N to 74°/N.

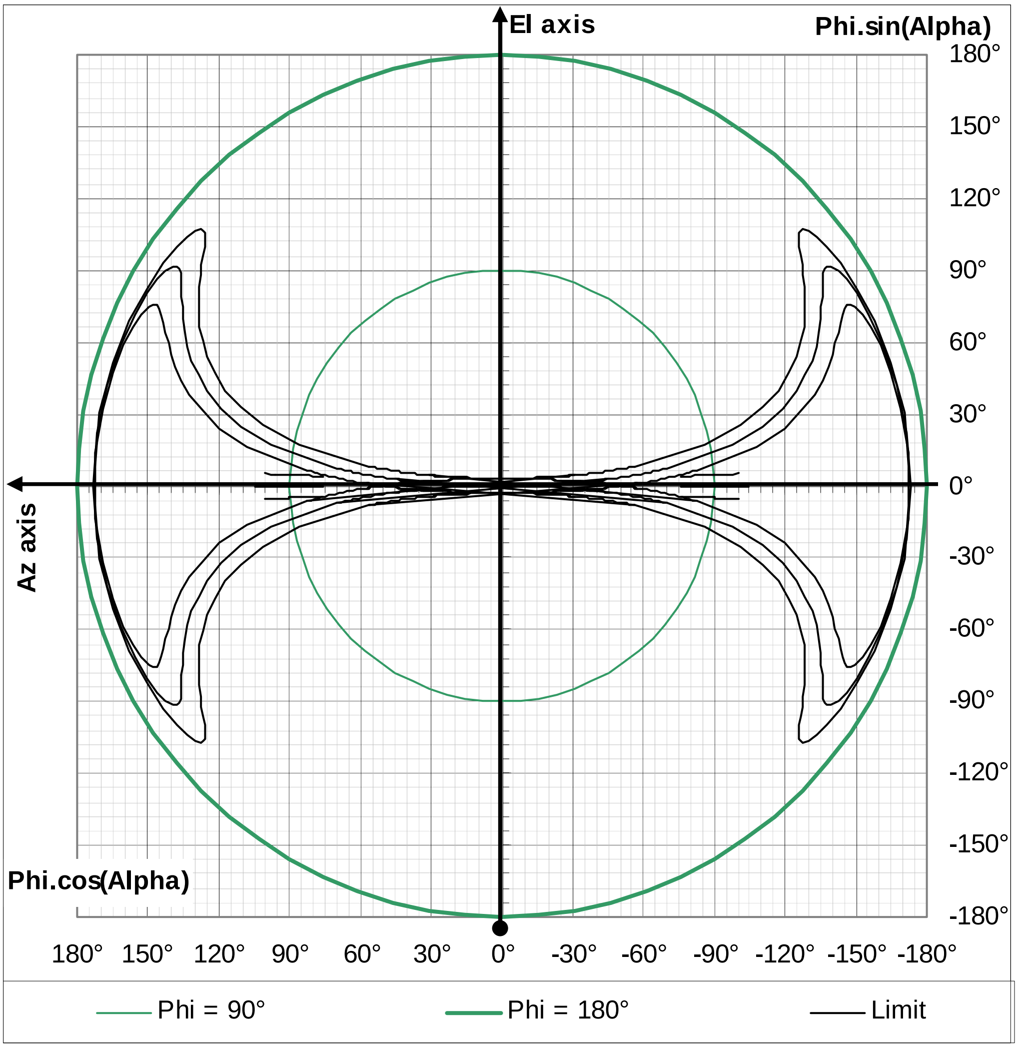

Figure 32: Limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from -74°/N to 74°/N

The above limit has been computed for 3 different latitudes of the adjacent satellite: -3°, 0° and +3°.

The above contour has been computed for 3 different latitudes of the adjacent satellite: -3°, 0° and +3°, using the method described for the Az-El antenna mount without GSO alignment.

With an alignment error or offset of the antenna Az-axis the contour is rotated as shown on figure 33 for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from -74°/N to 74°/N;
- an offset of the antenna Az-axis of +5°.

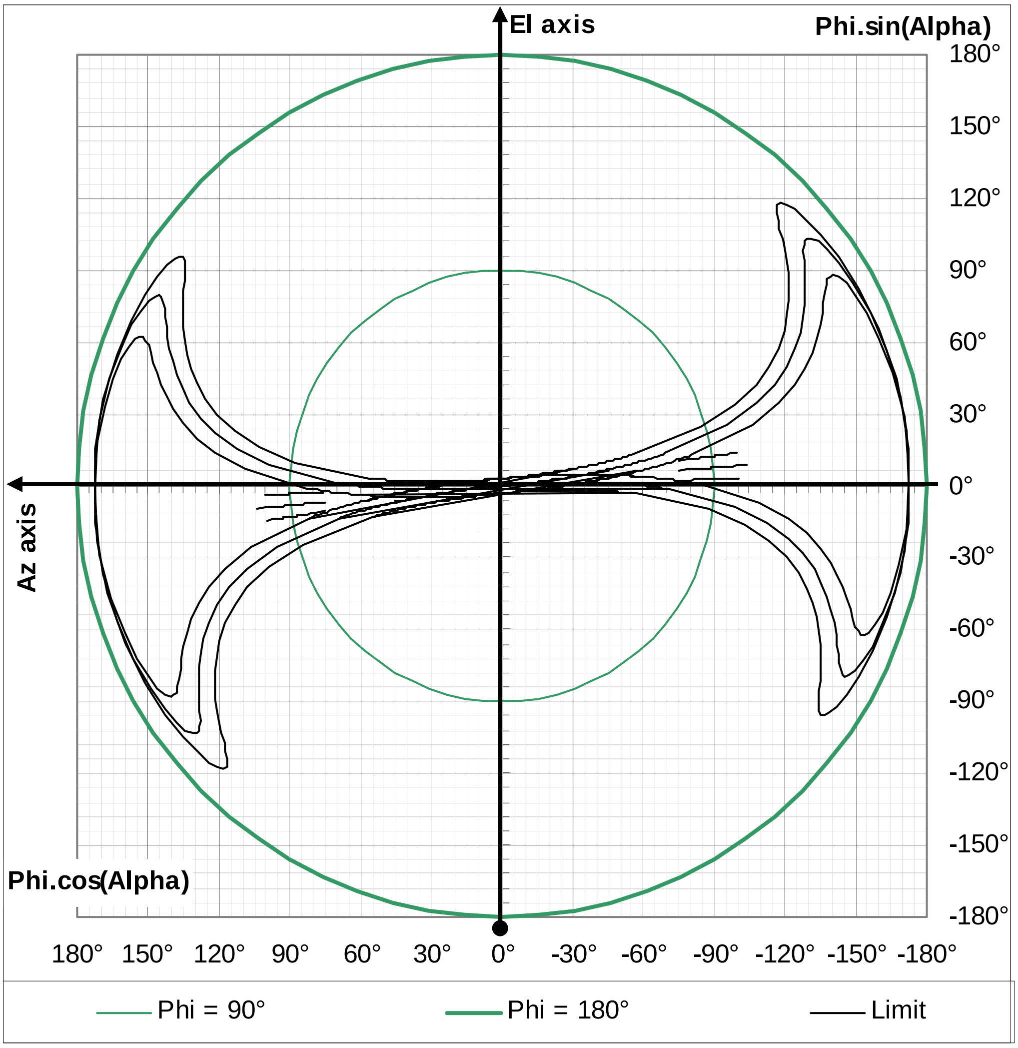

Figure 33: Limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from -74°/N to 74°/N and with an alignment error of 5°

It is obvious that a permanent offset i of the Az-axis result in a rotation of i the contour. This is verified on figure 33.

For antennas which could be used up-side down, the contour has to be computed for offset $i = 0°$ and for $i = 180°$.

It will be demonstrated that:

- For a antenna with an equatorial mount the maximum value Δi of the inclination error δi of the antenna azimuth axis $\overrightarrow{u_{Az,S}}$ due to an error $\delta El_{N,n}$ on the elevation and to an error $\delta Az_{N,n}$ on the azimuth of the estimated direction $\overrightarrow{\hat{N}}$ of the North pole in a station S_n at latitude Lt_n is given by the following relationship:

$$\sin\left(\frac{\Delta i}{2}\right)^2 = \sin\left(\frac{\delta El_{N,n}}{2}\right)^2 + \sin\left(\frac{\delta Az_{N,n}}{2}\right)^2 .\cos\left(Lt_n\right).\cos\left(Lt_n + \delta El_{N,n}\right) \qquad (5)$$

- When the latitude of the station is known with an accuracy highly better than Δi , e.g. with a GPS, then the following relationship applies:

$$\sin\left(\frac{\Delta i}{2}\right) = \sin\left(\left|\frac{\delta Az_{N,n}}{2}\right|\right).\cos\left(Lt_n\right) \qquad (6)$$

NOTE 1: A North-South error of 1 km on the location of the earth station corresponds to an error of 0,009° (= 360° x 1 km / 40 000 km) on the latitude of the earth station. This error is considered negligible for the antenna axis alignment. A GPS gives a better accuracy.

- For ESs designed to operate at any latitude, with the coordinates provided by a GPS, then the following relationship applies:

$$\sin\left(\frac{\Delta i}{2}\right)^2 = \sin\left(\frac{\delta El_{N,n}}{2}\right)^2 + \sin\left(\frac{\delta Az_{N,n}}{2}\right)^2 \qquad (7)$$

Figure 34 shows the contour for:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from -74°/N to 74°/N;
- no offset of the antenna Az-axis;
- an error $\delta Az_{N,n}$ on the azimuth of the estimated direction $\vec{\hat{N}}$ of the North pole in the station of 4°;
- an error $\delta El_{N,n}$ on the elevation of the estimated direction $\vec{\hat{N}}$ of the North pole in the station of 3°.

NOTE 2: These values correspond to a global error angle between the North pole direction and its estimation of 5° $\left(\sqrt{4^2+3^2}=5\right)$.

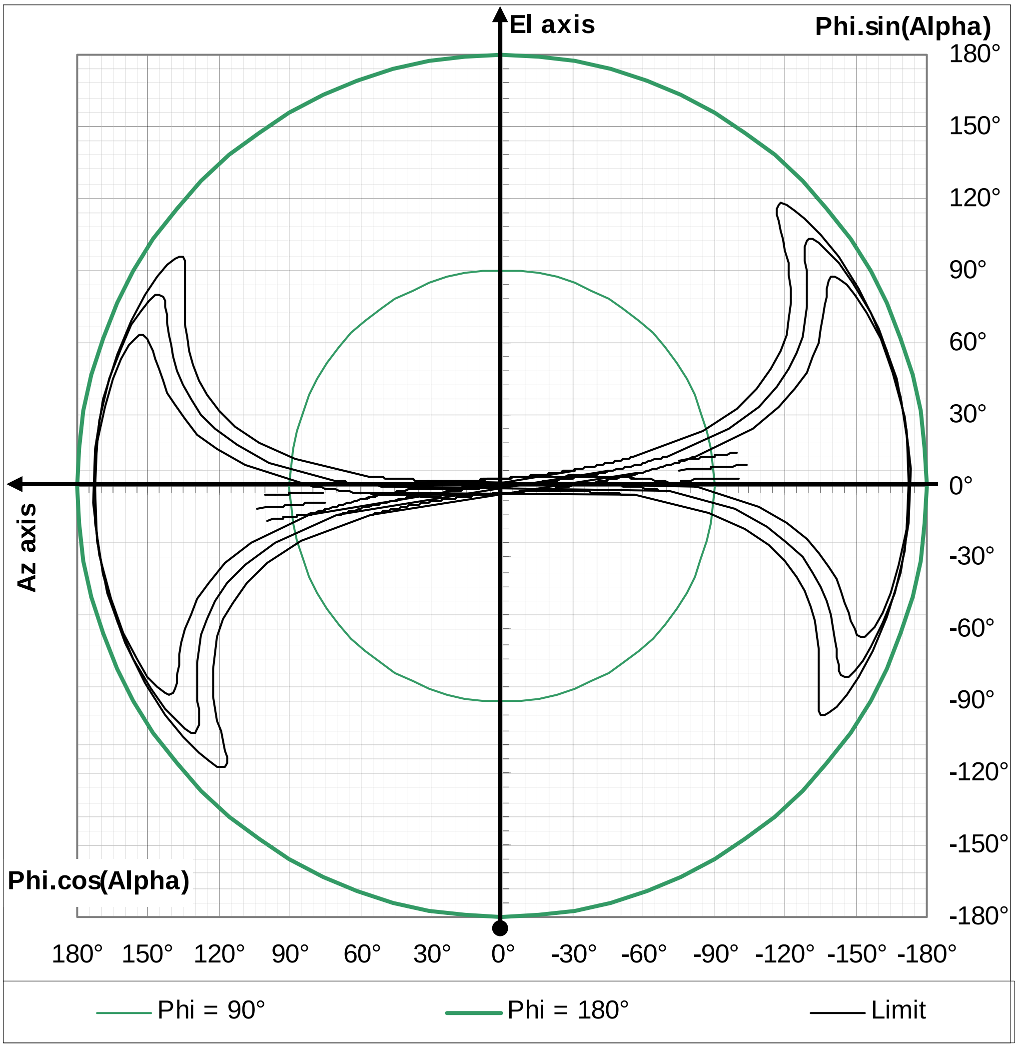

Figure 34: Limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from -74°/N to 74°/N and with a pole axis alignment error of 5°

5.4 Minimum longitude offset

The requirement for the protection of the other satellites of the GSO applies to adjacent satellites the longitudes of which are greater or equal to a minimum longitude offset (dLg_S_min).

To a minimum longitude offset (dLg_S_min) corresponds a minimum off-axis angle. The value of this minimum angle depends on the latitude of the ES and on the relative position of the pointed satellite.

On figure 35 the minimum off-axis angles are marked with red squares. Figure 35 corresponds to the case of alignment on the received electrical field:

- a minimum elevation angle of the pointed satellite of 7°;
- a minimum elevation angle of the satellites to be protected of 0°;
- ES latitudes from -74°/N to 74°/N;
- a tilt angle (i_E0) with the North pole direction of the electric field at the satellite of 22°;
- a longitude of the centre (Lg_c) of the satellite beam coverage of 5°/E;
- a latitude of the centre (Lt_c) of the satellite beam coverage of 50°/N;
- no offset of the antenna Az-axis;

- with alignment on the received H field or the E field but with an additional rotation of 90°;
- with alignment error of 1°;
- with a minimum longitude offset (dLg_S_min) of 3°.

A marker, represented by a dotted line, is used to determine the minimum size of the minimum off-axis angle. It is made of a straight line (Alpha = constant) a circle (Phi = constant) and a square with a side length equal to 2 × Phi and an inclination equal to Alpha.

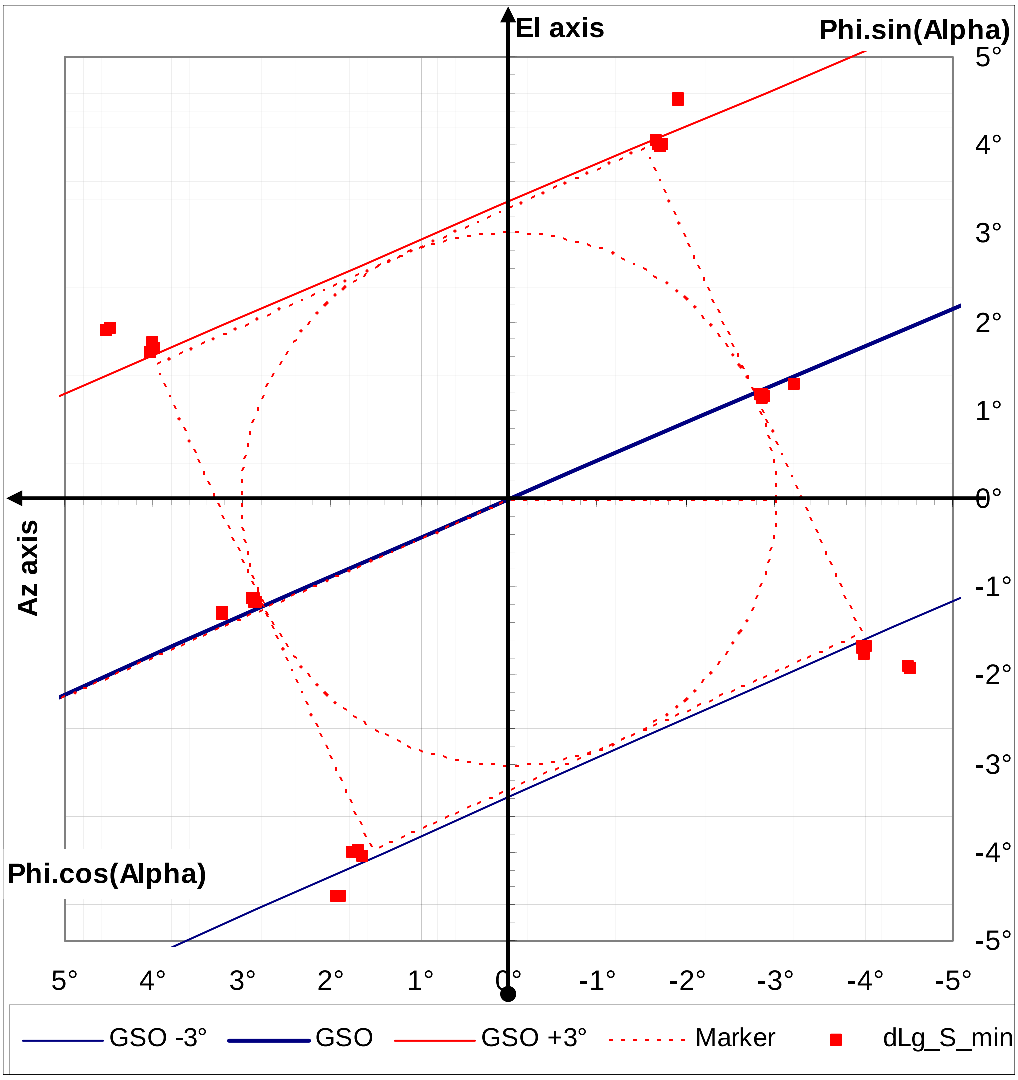

Figure 35: Minimum off-axis angle within the (α, φ) domain of the shadow of the GSO arc and its vicinity for latitudes from -74°/N to 74°/N, Lg_c = 5°, Lt_c = 50°, i_E0 = 22°, with alignment error of 1° and dLg_S_min = 3°

The minimum off-axis angles corresponding to the minimum longitude offset (dLg_S_min) have been computed for three adjacent satellite latitudes (-3°, 0° and +3°), and for following cases:

- the cases where the ES latitude varies from the minimum latitude to the maximum latitude, the ES antenna is pointed towards the satellite at the ES latitude, and the satellites to be protected are successively the East and West satellites at the minimum longitude offset (dLg_S_min);
- the cases where the ES latitude varies from the minimum latitude to the maximum latitude, the ES antenna is successively pointed towards the West and East satellites at the minimum elevation, and the satellites to be protected are successively the East and West satellites at the minimum longitude offset (dLg_S_min).

The smallest values of the minimum off-axis angles are obtained for the lowest elevation angles.

6 Mathematical analysis

6.1 Geographical coordinates

See figure 36.

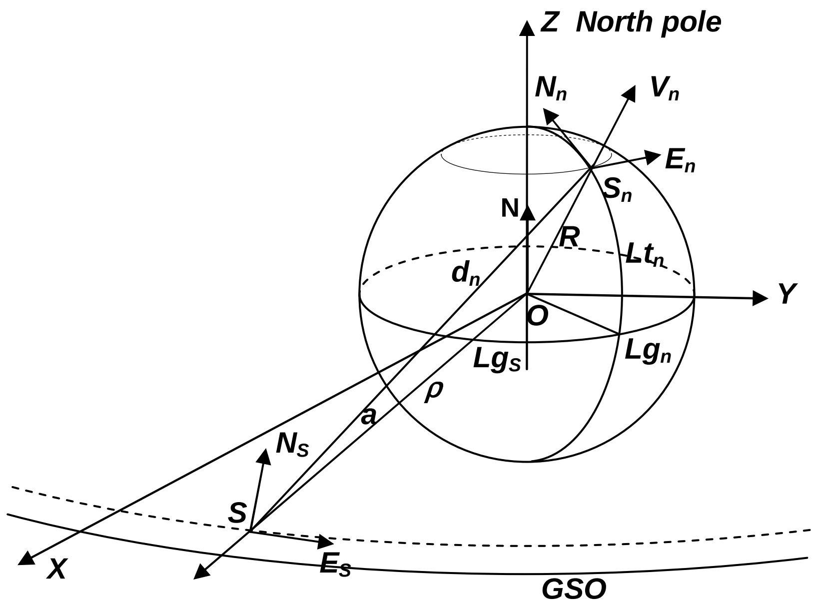

Figure 36: ES and satellite geographical coordinates

Let:

O: the centre of the Earth;

R: the mean Earth radius = 6 371 km (The Earth is assumed to be spherical.);

$\overrightarrow{OZ}$: the axis towards the North pole;

$\overrightarrow{OX}$: the axis at the intersection of the Greenwich median plane and the Earth equatorial plan;

$\overrightarrow{OY}$: the axis orthogonal to $\overrightarrow{OX}$ and $\overrightarrow{OZ}$ axis towards the East;

$\overrightarrow{N}$: the direction of the North pole.

Then:

$$\overrightarrow{N} = \begin{cases} X = 0 \\ Y = 0 \\ Z = 1 \end{cases} \qquad (8)$$

6.2 Earth station geographical coordinates

Let:

S_n: the earth station;

Lg_n: the earth station longitude, positive towards the East;

Lt_n: the earth station latitude, positive towards the North;

$\overrightarrow{V_n}$: the vertical at station S_n;

$\overrightarrow{N_n}$: the direction of the North at station S_n;

$\overrightarrow{E_n}$: the direction of the East at station S_n.

The Earth is assumed to be spherical.

Then:

$$\overrightarrow{S_n} = \begin{cases} X = R.\cos(Lt_n).\cos(Lg_n) \\ Y = R.\cos(Lt_n).\sin(Lg_n) \\ Z = R.\sin(Lt_n) \end{cases} \qquad (9)$$

$$\overrightarrow{V_n} = \begin{cases} X = \cos(Lt_n).\cos(Lg_n) \\ Y = \cos(Lt_n).\sin(Lg_n) \\ Z = \sin(Lt_n) \end{cases} \qquad (10)$$

$$\overrightarrow{N_n} = \begin{cases} X = -\sin(Lt_n).\cos(Lg_n) \\ Y = -\sin(Lt_n).\sin(Lg_n) \\ Z = \cos(Lt_n) \end{cases} \qquad (11)$$

$$\overrightarrow{E_n} = \begin{cases} X = -\sin(Lg_n) \\ Y = \cos(Lg_n) \\ Z = 0 \end{cases} \qquad (12)$$

$$\overrightarrow{S_n}.\overrightarrow{N_n} = 0 \qquad \overrightarrow{S_n}.\overrightarrow{E_n} = 0 \qquad \overrightarrow{S_n}.\overrightarrow{V_n} = \left\|\overrightarrow{S_n}\right\| = R \qquad (13)$$

6.3 Satellite geographical coordinates

Let:

S: the satellite on the GSO, or within its vicinity;

ρ: the nominal GSO radius = 42 164 km, and also radius the sphere of the GSO vicinity;

Lg_S: the satellite longitude, positive towards the East;

Lt_S: the satellite latitude, positive towards the North;

$\overrightarrow{N_S}$: the direction of the North at the satellite S position. This direction is different of the North Pole direction when the latitude of the satellite is not equal to 0;

$\overrightarrow{E_S}$: the direction of tangent to the GSO towards the East at the satellite S position.

Then:

$$\vec{S} = \begin{cases} X = \rho.\cos(Lt_S).\cos(Lg_S) \\ Y = \rho.\cos(Lt_S).\sin(Lg_S) \\ Z = \rho.\sin(Lt_S) \end{cases} \quad (14)$$

$$\overrightarrow{N_S} = \begin{cases} X = -\sin(Lt_S).\cos(Lg_S) \\ Y = -\sin(Lt_S).\sin(Lg_S) \\ Z = \cos(Lt_S) \end{cases} \quad (15)$$

$$\overrightarrow{E_S} = \begin{cases} X = -\sin(Lg_S) \\ Y = \cos(Lg_S) \\ Z = 0 \end{cases} \quad (16)$$

$$\vec{S}.\overrightarrow{N_S} = 0 \qquad \vec{S}.\overrightarrow{E_S} = 0 \quad (17)$$

6.4 Local coordinates

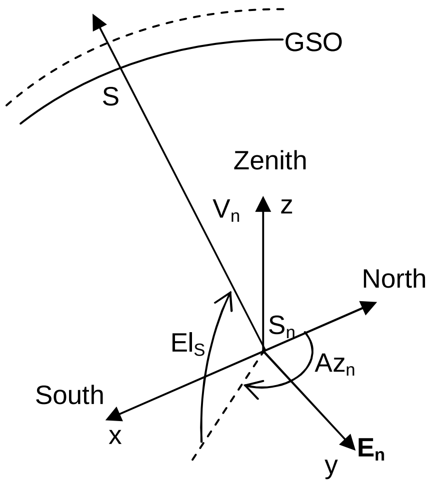

Figure 37: ES and satellite geographical coordinates

Let:

S_n : the earth station;

$\overrightarrow{Ox}$: the axis towards the local South at station S_n ;

NOTE: The GSO arc is mainly southwards for a station on the north hemisphere but it is northwards for a station on the south hemisphere.

$\overrightarrow{Oy}$: the axis towards the local East at station S_n ;

$\overrightarrow{Oz}$: the vertical axis at station S_n ;

Az_D: the azimuth of a considered direction $\overrightarrow{D}$;

El_D: the elevation of a considered direction $\overrightarrow{D}$;

$\overrightarrow{V_n}$: the vertical at station S_n ;

$\overrightarrow{N_n}$: the direction of the North at station S_n ;

$\overrightarrow{E_n}$: the direction of the East at station S_n ;

M : a point of the space with geographical coordinates $\left(X_M, Y_M, Z_M\right)$ and local coordinates $\left(x_M, y_M, z_M\right)$;

d_M the distance from S_n to M .

Then:

$$\overrightarrow{V_n} = \begin{cases} x = 0 \\ y = 0 \\ z = 1 \end{cases} \qquad \overrightarrow{N_n} = \begin{cases} x = -1 \\ y = 0 \\ z = 0 \end{cases} \qquad \overrightarrow{E_n} = \begin{cases} x = 0 \\ y = 1 \\ z = 0 \end{cases} \qquad d_M = \left\|\overrightarrow{S_n M}\right\| \tag{18}$$

$$\overrightarrow{S_n M} = \begin{cases} x_M = -d_M.\cos\left(El_M\right).\cos\left(Az_M\right) \\ y_M = d_M.\cos\left(El_M\right).\sin\left(Az_M\right) \\ z_M = d_M.\sin\left(El_M\right) \end{cases} \tag{19}$$

$$\overrightarrow{S_n M} = \begin{cases} x_M = -\overrightarrow{S_n M}.\overrightarrow{N_n} = -\left(\overrightarrow{M}.\overrightarrow{N_n} - \overrightarrow{S_n}.\overrightarrow{N_n}\right) & = -\overrightarrow{M}.\overrightarrow{N_n} \\ y_M = \overrightarrow{S_n M}.\overrightarrow{E_n} \quad = \quad \overrightarrow{M}.\overrightarrow{E_n} - \overrightarrow{S_n}.\overrightarrow{E_n} & = \overrightarrow{M}.\overrightarrow{E_n} \\ z_M = \overrightarrow{S_n M}.\overrightarrow{V_n} \quad = \quad \overrightarrow{M}.\overrightarrow{V_n} - \overrightarrow{S_n}.\overrightarrow{V_n} & = \overrightarrow{M}.\overrightarrow{V_n} - R \end{cases} \tag{20}$$

$$\overrightarrow{S_n M} = \begin{cases} x_M = X_M.\sin\left(Lt_n\right).\cos\left(Lg_n\right) + Y_M.\sin\left(Lt_n\right).\sin\left(Lg_n\right) - Z_M.\cos\left(Lt_n\right) \\ y_M = -X_M.\sin\left(Lg_n\right) + Y_M.\cos\left(Lg_n\right) \\ z_M = X_M.\cos\left(Lt_n\right).\cos\left(Lg_n\right) + Y_M.\cos\left(Lt_n\right).\sin\left(Lg_n\right) + Z_M.\sin\left(Lt_n\right) - R \end{cases} \tag{21}$$

$$\overrightarrow{S_nM} = \begin{cases} X_M = \begin{pmatrix} +x_M.\sin(Lt_n).\cos(Lg_n) - y_M.\sin(Lg_n) \\ +z_M.\cos(Lt_n).\cos(Lg_n) - R.\cos(Lt_n).\cos(Lg_n) \end{pmatrix} \\ Y_M = \begin{pmatrix} +x_M.\sin(Lt_n).\sin(Lg_n) + y_M.\cos(Lg_n) \\ +z_M.\cos(Lt_n).\sin(Lg_n) - R.\cos(Lt_n).\sin(Lg_n) \end{pmatrix} \\ Z_M = -x_M.\cos(Lt_n) + z_M.\sin(Lt_n) + R.\sin(Lt_n) \end{cases} \quad (22)$$

For any vector $\overrightarrow{AB} = \overrightarrow{S_nB} - \overrightarrow{S_nA}$ the above two sets of equations for the transformation of geographical coordinates into local coordinates and conversely are applicable for $R = 0$ due to the fact that the coordinates of $\overrightarrow{AB}$ are the differences of absolutes coordinates.

6.5 Satellite local coordinates

Let:

- S: the satellite on the GSO, or within its vicinity;
- ρ: the nominal GSO radius = 42 164 km, and also radius the sphere of the GSO vicinity;
- Az_S: the azimuth of the satellite S;
- El_S: the elevation of the satellite S;
- d_n: the distance from the station S_n to the satellite S;
- θ_n: the angle at the Earth centre between the direction of the satellite S and the direction of the station S_n;
- $\overrightarrow{N_S}$: the direction of the North at the satellite S position. This direction is different of the North Pole direction when the latitude of the satellite is not equal to 0;
- $\overrightarrow{E_S}$: the direction of tangent to the GSO towards the East at the satellite S position.

Then:

$$\cos(\theta_n) = \frac{\vec{S}.\overrightarrow{S_n}}{\|\vec{S}\|.\|\overrightarrow{S_n}\|} = \frac{\vec{S}.\overrightarrow{S_n}}{\rho.R} = \cos(Lt_S).\cos(Lt_n).\cos(Lg_S - Lg_n) + \sin(Lt_S).\sin(Lt_n) \quad (23)$$

$$d_n = \|\overrightarrow{S_nS}\| = \sqrt{\overrightarrow{S_nS}.\overrightarrow{S_nS}} = \sqrt{\vec{S}.\vec{S} + \overrightarrow{S_n}.\overrightarrow{S_n} - 2.\vec{S}.\overrightarrow{S_n}} = \sqrt{\rho^2 + R^2 - 2.\rho.R.\cos(\theta_n)} \quad (24)$$

$$\overrightarrow{S_nS} = \begin{cases} x_S = -\vec{S}.\overrightarrow{N_n} \\ y_S = +\vec{S}.\overrightarrow{E_n} \\ z_S = +\vec{S}.\overrightarrow{V_n} - R \end{cases} \quad (25)$$

$$\overrightarrow{S_nS} = \begin{cases} x_S = \rho.\left[\cos(Lt_S).\sin(Lt_n).\cos(Lg_S - Lg_n) - \sin(Lt_S).\cos(Lt_n)\right] \\ y_S = \rho.\left[\cos(Lt_S).\sin(Lg_S - Lg_n)\right] \\ z_S = \rho.\left[\cos(Lt_S).\cos(Lt_n).\cos(Lg_S - Lg_n) + \sin(Lt_S).\sin(Lt_n)\right] - R \end{cases} \quad (26)$$

$$\overrightarrow{S_nS} = \begin{cases} x_S = -d_n.\cos(El_S).\cos(Az_S) \\ y_S = d_n.\cos(El_S).\sin(Az_S) \\ z_S = d_n.\sin(El_S) \end{cases} \quad (27)$$

6.6 Other relationships between Satellite and ES coordinates

Let:

a_n: the aspect angle from the satellite S between the direction of the Earth centre O and the direction of the station S_n.

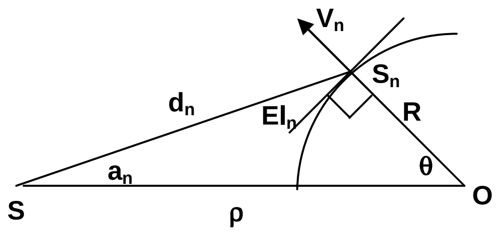

Figure 38: Triangle (0, S, S_n)

Within the triangle (0, S, S_n) the following relationships exist:

$$\frac{\sin\left(\frac{\pi}{2} + El_n\right)}{\rho} = \frac{\sin(\theta_n)}{d_n} = \frac{\sin(a_n)}{R} \quad \text{or:} \quad \frac{\cos(El_n)}{\rho} = \frac{\sin(\theta_n)}{d_n} = \frac{\sin(a_n)}{R} \quad (28)$$

$$\boxed{\cos(\theta_n) = \frac{\vec{S}.\vec{S}_n}{\rho.R} = \cos(Lt_S).\cos(Lt_n).\cos(Lg_S - Lg_n) + \sin(Lt_S).\sin(Lt_n)} \quad (29)$$

$$\boxed{d_n{}^2 = \rho^2 + R^2 - 2.\rho.R.\cos(\theta_n)} \quad (30)$$

Then for a given satellite S and a given earth station S_n the elevation El_n of the satellite in station S_n may be determined with the following formulae:

$$\boxed{\sin(El_n) = \frac{\overrightarrow{S_nS}.\vec{V}_n}{\left\|\overrightarrow{S_nS}\right\|} = \frac{\rho.\cos(\theta_n) - R}{\sqrt{\rho^2 + R^2 - 2.\rho.R.\cos(\theta_n)}}} \quad (31)$$

$$\cos(El_n).\cos(Az_n) = \frac{\overrightarrow{S_nS}.\vec{N}_n}{\left\|\overrightarrow{S_nS}\right\|} = -\frac{x_S}{d_n} = -\frac{\rho.\begin{pmatrix} +\sin(Lt_n).\cos(Lt_S).\cos(Lg_S - Lg_n) \\ -\cos(Lt_n).\sin(Lt_S) \end{pmatrix}}{\sqrt{\rho^2 + R^2 - 2.\rho.R.\cos(\theta)}} \quad (32)$$

$$\cos(El_n).\sin(Az_n) = \frac{\overrightarrow{S_nS}.\vec{E}_n}{\left\|\overrightarrow{S_nS}\right\|} = \frac{y_S}{d_n} = \frac{\rho.\cos(Lt_S).\sin(Lg_S - Lg_n)}{\sqrt{\rho^2 + R^2 - 2.\rho.R.\cos(\theta)}} \tag{33}$$

$$Az_n = ArcTan2\left(\frac{y_S}{d_n}, -\frac{x_S}{d_n}\right) = ArcTan2(y_S, -x_S) \tag{34}$$

with:

$$ArcTan2(a.\sin(x), a.\cos(x)) = x \qquad for \qquad a > 0 \quad and \qquad x \in [-\pi, +\pi[\tag{35}$$

$$\boxed{Az_n = ArcTan2\left(\left(\sin(Lg_S - Lg_n)\right), \left(\begin{array}{l} -\sin(Lt_n).\cos(Lg_S - Lg_n) \\ +\tan(Lt_S).\cos(Lt_n) \end{array}\right)\right)} \tag{36}$$

The satellite longitude Lg_S corresponding to a given elevation El_n is a solution of the following equations:

$$\begin{cases} \sin(El_n) = \dfrac{\rho.\cos(\theta_n) - R}{\sqrt{\rho^2 + R^2 - 2.\rho.R.\cos(\theta_n)}} \\ \cos(\theta_n) = \cos(Lt_S).\cos(Lt_n).\cos(Lg_S - Lg_n) + \sin(Lt_S).\sin(Lt_n) \end{cases} \tag{37}$$

or:

$$\begin{cases} \rho^2.\cos(\theta_n)^2 - 2.\left(\rho.R.\cos(El_n)^2\right).\cos(\theta_n) - \left(\rho^2.\sin(El_n)^2 - R^2.\cos(El_n)^2\right) = 0 \\ \cos(Lg_S - Lg_n) = \dfrac{\cos(\theta_n) - \sin(Lt_S).\sin(Lt_n)}{\cos(Lt_S).\cos(Lt_n)} \end{cases} \tag{38}$$

The discriminant Δ of the above 2nd degree equation of $\cos(\theta_n)$ is:

$$\Delta = 4.\rho^2.\sin(El_n)^2.\left(\rho^2 + R^2.\cos(El_n)^2\right) \tag{39}$$

Two values of $\cos(\theta_n)$ are obtained:

$$\cos(\theta_n) = \left(\frac{R}{\rho}\right).\cos(El_n)^2 \pm \sin(El_n).\sqrt{1 - \left(\frac{R}{\rho}\right)^2.\cos(El_n)^2} \tag{40}$$

The larger values of $\cos(\theta_n)$ corresponds to an earth station visible from the satellite, the other value to an earth station hidden by the Earth as shown on figure 39.

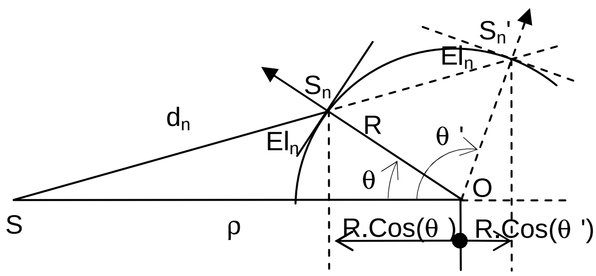

Figure 39: Triangle (0, S, S_n)

Then:

$$\begin{cases} \cos(\theta_n) = \left(\dfrac{R}{\rho}\right).\cos(El_n)^2 + \sin(El_n).\sqrt{1-\left(\dfrac{R}{\rho}\right)^2.\cos(El_n)^2} \\ \cos(Lg_S - Lg_n) = \dfrac{\cos(\theta_n) - \sin(Lt_S).\sin(Lt_n)}{\cos(Lt_S).\cos(Lt_n)} \end{cases} \quad (41)$$

The maximum satellite latitude $Lt_{n,\,max}$ corresponding to a given elevation El_n is solution of the following equations:

$$\begin{cases} \cos(\theta_n) = \left(\dfrac{R}{\rho}\right).\cos(El_n)^2 + \sin(El_n).\sqrt{1-\left(\dfrac{R}{\rho}\right)^2.\cos(El_n)^2} \\ \cos(Lg_S - Lg_n) = \dfrac{\cos(\theta_n) - \sin(Lt_S).\sin(Lt_{n,\max})}{\cos(Lt_S).\cos(Lt_{n,\max})} \\ Lg_S = Lg_n \end{cases} \quad (42)$$

or:

$$\cos(Lt_S - Lt_{n,\max}) = \left(\frac{R}{\rho}\right).\cos(El_n)^2 + \sin(El_n).\sqrt{1-\left(\frac{R}{\rho}\right)^2.\cos(El_n)^2} \quad (43)$$

6.7 Antenna Cartesian coordinates

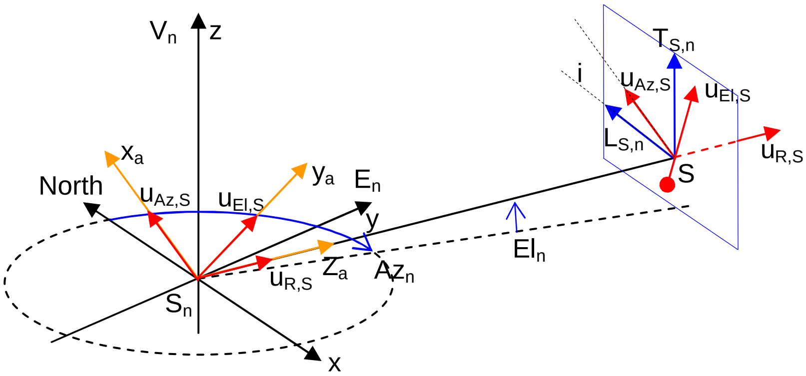

Figure 40: Antenna Cartesian coordinates

Let:

S_n : the earth station with its antenna;

$\overrightarrow{V_n}$: the vertical at station S_n ;

S: the satellite;

$\overrightarrow{u_{R,S}}$: the antenna main beam axis, or radial axis towards the satellite;

$\overrightarrow{u_{Az,S}}$: the antenna azimuth axis (Az-axis);

$\overrightarrow{u_{El,S}}$: the antenna elevation axis (El-axis);

$\overrightarrow{L_{S,n}}$: the left hand side direction at the satellite for station S_n : orthogonal to $\overrightarrow{S_nS}$ and $\overrightarrow{V_n}$;

$\overrightarrow{T_{S,n}}$: the direction at the satellite within the vertical plane in S_n containing S, orthogonal to $\overrightarrow{S_nS}$ and to $\overrightarrow{L_{S,n}}$, and oriented towards the antenna top;

i: The inclination angle between the antenna azimuth axis (Az-axis) $\overrightarrow{u_{Az,S}}$ and the satellite left direction $\overrightarrow{L_{S,n}}$;

(x, y, z) the earth station local coordinates;

(x_a, y_a, z_a) the antenna coordinates;

$\overrightarrow{M}$: a point of the space with local coordinates (x_M, y_M, z_M) or (El_M, Az_M) and antenna coordinates $(x_{a,M}, y_{a,M}, z_{a,M})$.

Then:

$$\overrightarrow{u_{R,S}} = \frac{\overrightarrow{S_nS}}{\left\|\overrightarrow{S_nS}\right\|} = \begin{cases} x = -\cos(El_S).\cos(Az_S) \\ y = \cos(El_S).\sin(Az_S) \\ z = \sin(El_S) \end{cases} \quad (44)$$

$$\overrightarrow{L_{S,n}} = \left(\frac{\overrightarrow{V_n} \wedge \overrightarrow{S_nS}}{\left\|\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right\|} \right) = \begin{cases} x = -\sin(Az_S) \\ y = -\cos(Az_S) \\ z = 0 \end{cases} \quad (45)$$

$$\overrightarrow{T_{S,n}} = \left(\frac{\overrightarrow{S_nS}}{\left\|\overrightarrow{S_nS}\right\|} \right) \wedge \overrightarrow{L_{S,n}} = \begin{cases} x = +\sin(El_S).\cos(Az_S) \\ y = -\sin(El_S).\sin(Az_S) \\ z = +\cos(El_S) \end{cases} \quad (46)$$

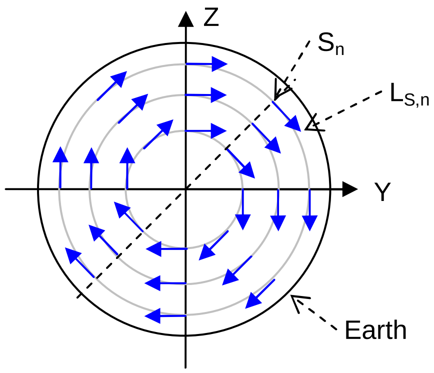

Figure 41: Satellite left hand side direction $\overrightarrow{L_{S,n}}$ for a station S_n, seen from the satellite

As shown on figure 41, the satellite left hand side direction for a ES S_n is towards the East when the ES is located in the north hemisphere and in the meridian plan of the satellite, otherwise any other direction is possible. The variations of that direction are continuous except at the sub-satellite point on the Earth along any line passing by that point and where the discontinuity at this point is equal to ±180°. The use of the satellite left hand side direction for a ES S_n is suitable for antennas which are rotated by 180° in azimuth when crossing the equator from one position to the position symmetrical to the sub-satellite point and when pointing the same satellite, and:

$$\overrightarrow{u_{Az,S}} = \cos(i).\overrightarrow{L_{S,n}} + \sin(i).\overrightarrow{T_{S,n}} = \begin{cases} x = -\cos(i).\sin(Az_S) + \sin(i).\sin(El_S).\cos(Az_S) \\ y = -\cos(i).\cos(Az_S) - \sin(i).\sin(El_S).\sin(Az_S) \\ z = \quad +\sin(i).\cos(El_S) \end{cases} \quad (47)$$

$$\overrightarrow{u_{El,S}} = -\sin(i).\overrightarrow{L_{S,n}} + \cos(i).\overrightarrow{T_{S,n}} = \begin{cases} x = +\sin(i).\sin(Az_S) + \cos(i).\sin(El_S).\cos(Az_S) \\ y = +\sin(i).\cos(Az_S) - \cos(i).\sin(El_S).\sin(Az_S) \\ z = \quad +\cos(i).\cos(El_S) \end{cases} \quad (48)$$

$$\overrightarrow{S_nM} = \begin{cases} x = -\cos(El_M).\cos(Az_M) \\ y = \cos(El_M).\sin(Az_M) \\ z = \sin(El_M) \end{cases} \quad (49)$$

$$x_{a,M} = \overrightarrow{S_nM}.\overrightarrow{u_{Az,S}} = \begin{pmatrix} +\sin(i).\sin(El_M).\cos(El_S) \\ +\cos(i).\cos(El_M).\sin(Az_S - Az_M) \\ -\sin(i).\cos(El_M).\sin(El_S).\cos(Az_S - Az_M) \end{pmatrix} \quad (50)$$

$$y_{a,M} = \overrightarrow{S_nM}.\overrightarrow{u_{El,S}} = \begin{pmatrix} -\sin(i).\cos(El_M).\sin(Az_S - Az_M) \\ -\cos(i).\sin(El_S).\cos(El_M).\cos(Az_S - Az_M) \\ +\cos(i).\cos(El_S).\sin(El_M) \end{pmatrix} \quad (51)$$

$$z_{a,M} = \overrightarrow{S_nM}.\overrightarrow{u_{R,S}} = \cos(El_S).\cos(El_M).\cos(Az_S - Az_M) + \sin(El_S).\sin(El_M) \quad (52)$$

6.8 Antenna polar coordinates

Two types of polar coordinates are used:

- one is used to describe the North-South and East-West variations (φ_{El} and φ_{Az}) from the main beam axis as it could be seen by an observer at the antenna location;
- the other describe the antenna radiation patterns, with the angle α of the plan containing the main beam axis and the direction to the considered point M and the off-axis angle φ of the direction of M from the main beam axis.

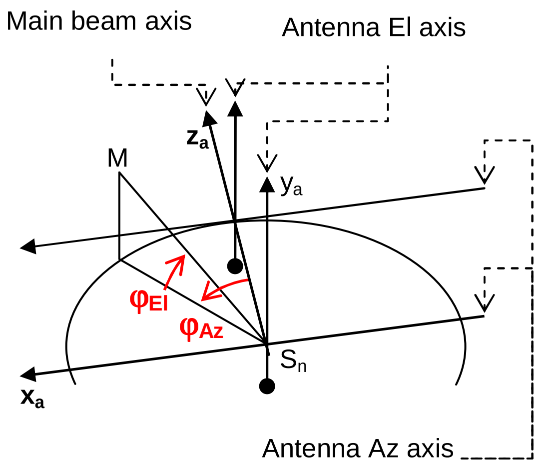

Figure 42: Off-axis angles φ_{El} and φ_{Az} of a direction $\overrightarrow{S_n M}$

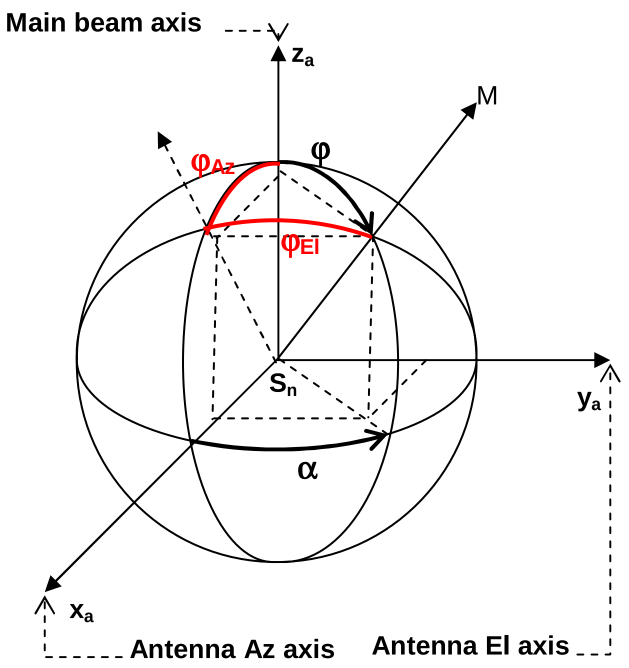

Figure 43: Off-axis angles (φ), and α angle of a direction $\overrightarrow{S_n M}$

Let:

S_n : the earth station with its antenna;

M : a point of the space (El_M, Az_M) and antenna coordinates;

α_M: the angle of the projection of the direction of M on the plane orthogonal to the antenna main beam axis with the antenna azimuth axis (Az-axis); and

φ_M: the off-axis angle of the direction of M with the antenna main beam direction;

φ_{El}: the angle of the direction of M with the plane defined by the antenna main beam axis and the antenna azimuth axis (Az-axis);

φ_{Az}: the angle of the projection of the direction of M on the plane define by the antenna main beam axis and the antenna azimuth axis (Az-axis) with the antenna main beam axis.

Then:

$$\overrightarrow{S_n M} = \begin{cases} x_a = \cos(\varphi_{El}).\sin(\varphi_{Az}) \\ y_a = \sin(\varphi_{El}) \\ z_a = \cos(\varphi_{El}).\cos(\varphi_{Az}) \end{cases} \quad (53)$$

$$\overrightarrow{S_n M} = \begin{cases} x_a = \sin(\varphi_M).\cos(\alpha_M) \\ y_a = \sin(\varphi_M).\sin(\alpha_M) \\ z_a = \cos(\varphi_M) \end{cases} \quad (54)$$

and:

$$\varphi_{El} = ArcSin(y_a) \quad (55)$$

$$\varphi_{Az} = ArcTan2(x_a, z_a) \quad (56)$$

$$\varphi_M = ArcCos(z_{a,M}) = ArcCos(\cos(\varphi_{El}).\cos(\varphi_{Az})) \quad (57)$$

$$\alpha_M = \begin{cases} = ArcTan2\left(\dfrac{y_a}{\sin(\varphi_M)}, \dfrac{x_a}{\sin(\varphi_M)}\right) \\ = ArcTan2(y_a, x_a) + \pi.\Upsilon(-\sin(\varphi_M))\ rad \\ = ArcTan2(\sin(\varphi_{El}), \cos(\varphi_{El}).\sin(\varphi_{Az})) + \pi.\Upsilon(-\sin(\varphi_M))\ rad \end{cases} \quad (58)$$

$$0° \le \varphi_M \le 180° \qquad \Rightarrow \qquad 0 \le \sin(\varphi_{Az}) \le 1 \quad (59)$$

$$\alpha_M = ArcTan2(\sin(\varphi_{El}), \cos(\varphi_{El}).\sin(\varphi_{Az})) \quad (60)$$

6.9 Antenna Az-axis alignment

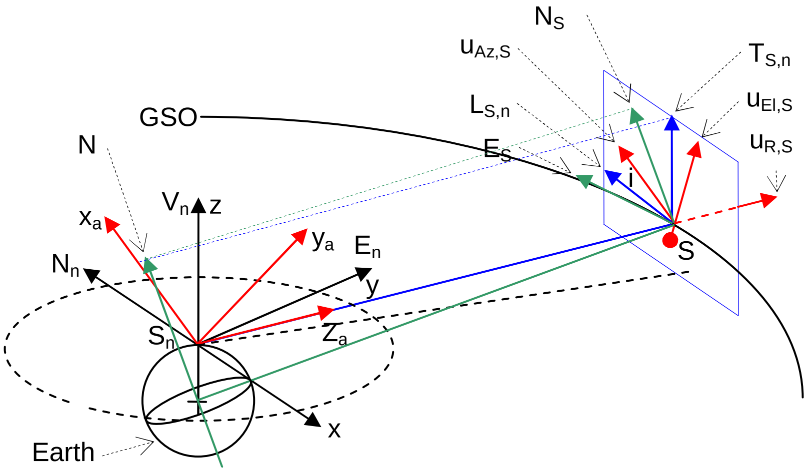

Figure 44a: Antenna Cartesian coordinates

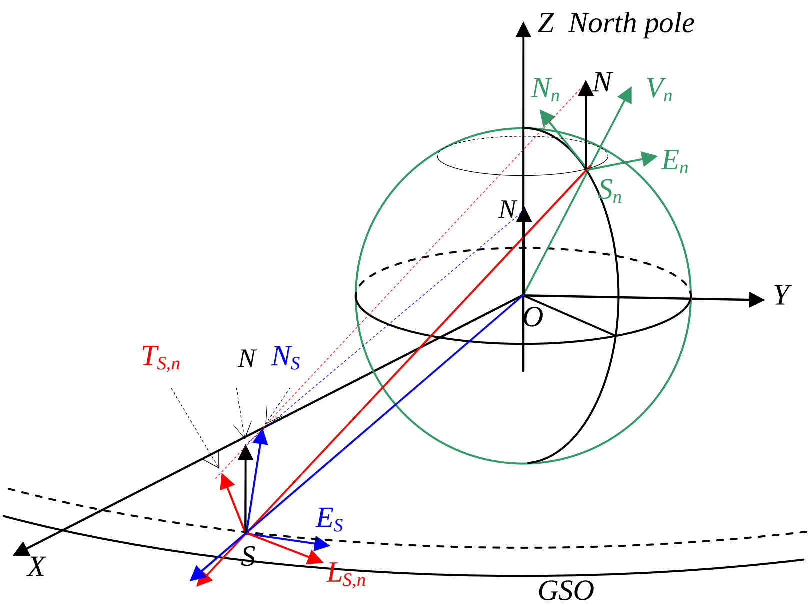

Figure 44b: Antenna Cartesian coordinates

Let:

S_n: the earth station with its antenna;

$\overrightarrow{V}_n$: the vertical at station S_n;

$\overrightarrow{N}_n$: the direction of the North at station S_n;

$\overrightarrow{E}_n$: the direction of the East at station S_n;

S: the satellite;

$\overrightarrow{N}_S$: the direction of the North at the satellite S position and orthogonal to $\overline{OS}$;

$\overrightarrow{E_S}$: the direction of tangent to the GSO towards the East at the satellite S position, orthogonal to $\overrightarrow{OS}$ and $\overrightarrow{N_S}$;

$\overrightarrow{u_{R,S}}$: the antenna main beam axis, or radial axis towards the satellite;

$\overrightarrow{u_{Az,S}}$: the antenna azimuth axis (Az-axis);

$\overrightarrow{u_{El,S}}$: the axis antenna elevation axis (El-axis);

$\overrightarrow{L_{S,n}}$: the left hand side direction at the satellite for station S_n, orthogonal to $\overrightarrow{S_nS}$ and $\overrightarrow{V_n}$;

$\overrightarrow{T_{S,n}}$: the direction at the satellite within the vertical plane in S_n containing S, orthogonal to $\overrightarrow{S_nS}$ and to $\overrightarrow{L_{S,n}}$, and oriented towards the antenna top;

i: the inclination angle between the antenna azimuth axis (Az-axis) $\overrightarrow{u_{Az,S}}$ and the left hand side direction $\overrightarrow{L_{S,n}}$ of the satellite for station #n;

i_{Az}: the inclination angle between the $\overrightarrow{u_{Az,S}}$ and $\overrightarrow{L_{S,n}}$ or between the $\overrightarrow{u_{El,S}}$ and $\overrightarrow{T_{S,n}}$ with the antenna mount;

(x, y, z) the earth station local coordinates;

(x_a, y_a, z_a) the antenna coordinates.

Then:

$$\overrightarrow{u_{El,S}} \quad \frac{\overrightarrow{E_S} \wedge \overrightarrow{S_nS}}{\left\| \overrightarrow{E_S} \wedge \overrightarrow{S_nS} \right\|} \qquad (61)$$

$$i_S = angle\left(\overrightarrow{u_{El,S}}, \overrightarrow{T_{S,n}}\right) \qquad (62)$$

The values of $\sin\left(i_S\right)$ may be obtained from the following set of equations:

$$\overrightarrow{S_nS} = \begin{cases} x_S = \rho.\left[\cos\left(Lt_S\right).\sin\left(Lt_n\right).\cos\left(Lg_S - Lg_n\right) - \sin\left(Lt_S\right).\cos\left(Lt_n\right)\right] \\ y_S = \rho.\left[\cos\left(Lt_S\right).\sin\left(Lg_S - Lg_n\right)\right] \\ z_S = \rho.\left[\cos\left(Lt_S\right).\cos\left(Lt_n\right).\cos\left(Lg_S - Lg_n\right) + \sin\left(Lt_S\right).\sin\left(Lt_n\right)\right] - R \end{cases} \qquad (63)$$

$$\overrightarrow{V_n} = \begin{cases} x = 0 \\ y = 0 \\ z = 1 \end{cases} \qquad (64)$$

$$\overrightarrow{L_{S,n}} = \left(\frac{\overrightarrow{V_n} \wedge \overrightarrow{S_nS}}{\left\| \overrightarrow{V_n} \wedge \overrightarrow{S_nS} \right\|} \right) \qquad (65)$$

$$\overrightarrow{T_{S,n}} = \left(\frac{\overrightarrow{S_n S}}{\left\| \overrightarrow{S_n S} \right\|} \right) \wedge \overrightarrow{L_{S,n}} \qquad (66)$$

$$\overrightarrow{E_S} = \begin{cases} x = -\sin(Lt_n).\sin(Lg_S - Lg_n) \\ y = \cos(Lg_S - Lg_n) \\ z = -\cos(Lt_n).\sin(Lg_S - Lg_n) \end{cases} \qquad (67)$$

$$\overrightarrow{u_{El,S}} \quad \frac{\overrightarrow{E_S} \wedge \overrightarrow{S_n S}}{\left\| \overrightarrow{E_S} \wedge \overrightarrow{S_n S} \right\|} \qquad (68)$$

$$i_{Az} = angle\left(\overrightarrow{u_{El,S}}, \overrightarrow{T_{S,n}}\right) \qquad (69)$$

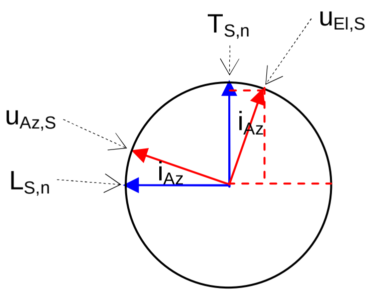

Figure 45: Projections of $\overrightarrow{u_{El,S}}$ on $\overrightarrow{L_{S,n}}$ and $\overrightarrow{T_{S,n}}$

$$\begin{cases} \cos(i_{Az}) = \overrightarrow{u_{El,S}}.\overrightarrow{T_{S,n}} \\ \sin(i_{Az}) = -\overrightarrow{u_{El,S}}.\overrightarrow{L_{S,n}} \end{cases} \Rightarrow \quad i_{Az} = ArcTan2\left(\sin(i_{Az}), \cos(i_{Az})\right) \qquad (70)$$

The antenna azimuth axis (Az-axis) is aligned with the tangent to the GSO arc at the satellite S position when: $i = -i_{Az}$.

6.10 Az-axis alignment of an antenna with a fixed polarizer

Let:

- S: the satellite;
- S_c : the centre of the satellite beam coverage on the Earth;
- S_n : the earth station with its antenna;
- $\overrightarrow{V_n}$: the vertical at station S_n ;
- $\overrightarrow{N}$: the direction of the North pole;
- $\overrightarrow{u_n}$: the direction of station S_n from the satellite;
- $\overrightarrow{u_c}$: the direction of the centre S_c of the satellite beam coverage on the Earth from the satellite;

$\overrightarrow{L_{S,n}}$: the left hand side direction at the satellite for station S_n, orthogonal to $\overrightarrow{S_nS}$ and $\overrightarrow{V_n}$;

$\overrightarrow{E_{S,c}}$: the direction towards the East at the satellite: orthogonal to $\overrightarrow{SS_c}$ and $\overrightarrow{N}$;

$\overrightarrow{E_0}$: the radiated electric field at the satellite;

$\overrightarrow{H_0}$: the radiated magnetic field at the satellite;

Z_0 : the vacuum impedance: $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120.\pi = 376,6\,\Omega$;

i_{E_0} : the tilt angle of radiated electric field at the satellite relative to the North pole direction $\overrightarrow{N}$;

$\overrightarrow{E_{r,n}}$: the received electric field at station #n;

$\overrightarrow{H_{r,n}}$: the received magnetic field at station #n;

i: the inclination angle between the antenna azimuth axis (Az-axis) $\overrightarrow{u_{Az,S}}$ and the left hand side direction $\overrightarrow{L_{S,n}}$ of the satellite for station #n;

i_E: the inclination angle between the received electric field $\overrightarrow{E_{r,n}}$ or magnetic field $\overrightarrow{H_{r,n}}$ and $\overrightarrow{L_{S,n}}$.

Then:

$$\overrightarrow{u_n} \quad \frac{\overrightarrow{SS_n}}{\left\|\overrightarrow{SS_n}\right\|} \tag{71}$$

$$\overrightarrow{u_c} \quad \frac{\overrightarrow{SS_c}}{\left\|\overrightarrow{SS_c}\right\|} \tag{72}$$

$$\overrightarrow{L_{S,n}} = \frac{\overrightarrow{SS_n} \wedge \overrightarrow{V_n}}{\left\|\overrightarrow{SS_n} \wedge \overrightarrow{V_n}\right\|} \tag{73}$$

$$\overrightarrow{E_{S,c}} = \frac{\overrightarrow{SS_c} \wedge \overrightarrow{N}}{\left\|\overrightarrow{SS_c} \wedge \overrightarrow{N}\right\|} \tag{74}$$

$$\overrightarrow{E_0} = \cos\left(i_{E_0}\right).\overrightarrow{N} + \sin\left(i_{E_0}\right).\overrightarrow{E_{S,c}} \tag{75}$$

$$Z_0.\overrightarrow{H_0} = \overrightarrow{u_c} \wedge \overrightarrow{E_0} \tag{76}$$

$$\overrightarrow{E_{r,n}} = \frac{\overrightarrow{u_n} \wedge \left(\overrightarrow{E_0} \wedge \overrightarrow{u_n}\right) + Z_0.\left(\overrightarrow{H_0} \wedge \overrightarrow{u_n}\right)}{\left\|\overrightarrow{u_n} \wedge \left(\overrightarrow{E_0} \wedge \overrightarrow{u_n}\right) + Z_0.\left(\overrightarrow{H_0} \wedge \overrightarrow{u_n}\right)\right\|} \tag{77}$$

$$Z_0.\overrightarrow{H_{r,n}} = \overrightarrow{u_n} \wedge \overrightarrow{E_{r,n}} \tag{78}$$

When the antenna Az-axis is aligned on the received electric field:

$$\begin{cases} \overrightarrow{u_{Az,S}} = \overrightarrow{E_{r,n}} \\ \overrightarrow{u_{El,S}} = -\overrightarrow{H_{r,n}} \end{cases} \Rightarrow \begin{cases} \cos\left(i_E + \frac{\pi}{2}\right) = \overrightarrow{L_{S,n}}.\overrightarrow{u_{Az,S}} = \overrightarrow{L_{S,n}}.\overrightarrow{E_{r,n}} = -\sin\left(i_E\right) \\ \sin\left(i_E + \frac{\pi}{2}\right) = \overrightarrow{T_{S,n}}.\overrightarrow{u_{Az,S}} = \overrightarrow{T_{S,n}}.\overrightarrow{E_{r,n}} = \cos\left(i_E\right) \end{cases} \quad (79)$$

When the antenna Az-axis is aligned on the received magnetic field:

$$\begin{cases} \overrightarrow{u_{Az,S}} = \overrightarrow{H_{r,n}} \\ \overrightarrow{u_{El,S}} = \overrightarrow{E_{r,n}} \end{cases} \Rightarrow \begin{cases} \cos\left(i_E\right) = \overrightarrow{L_{S,n}}.\overrightarrow{u_{Az,S}} = \overrightarrow{L_{S,n}}.\overrightarrow{H_{r,n}} \\ \sin\left(i_E\right) = \overrightarrow{T_{S,n}}.\overrightarrow{u_{Az,S}} = \overrightarrow{T_{S,n}}.\overrightarrow{H_{r,n}} \end{cases} \quad (80)$$

$$i_E = ArcTan2\left(\sin\left(i_E\right), \cos\left(i_E\right)\right) \quad (81)$$

The antenna azimuth axis (Az-axis) is aligned with the received electric field $\overrightarrow{E_{r,n}}$ or magnetic field $\overrightarrow{H_{r,n}}$ from satellite S when: $i = -i_E$.

6.11 Antenna with equatorial mount

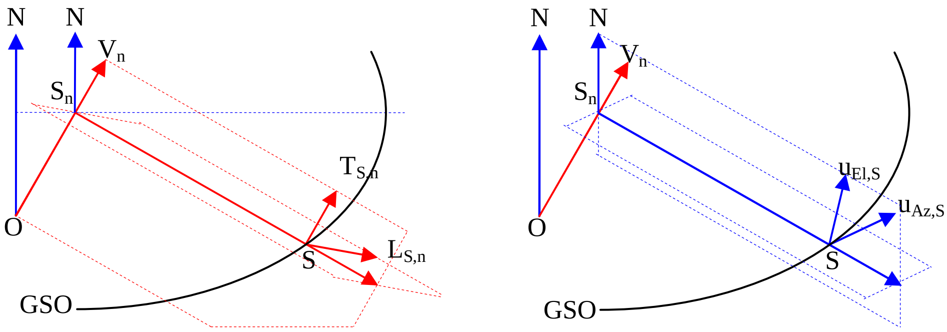

Figure 46: Antenna Cartesian coordinate systems relative to the local vertical direction and relative to the Pole axis

Let:

S_n : the earth station with its antenna;

$\overrightarrow{V_n}$: the vertical at station S_n ;

S: the satellite;

$\overrightarrow{u_{Az,S}}$: the antenna azimuth axis (Az-axis);

$\overrightarrow{u_{El,S}}$: the axis antenna elevation axis (El-axis);

$\overrightarrow{N}$: the direction of the North;

$\overrightarrow{N_S}$: the direction of the North at the satellite S position and orthogonal to $\overline{OS}$;

$\overrightarrow{E_S}$: the direction of tangent to the GSO towards the East at the satellite S position, orthogonal to $\overrightarrow{OS}$ and $\overrightarrow{N_S}$;

$\overrightarrow{L_{S,n}}$: the left hand side direction at the satellite for station S_n : orthogonal to $\overrightarrow{S_nS}$ and $\overrightarrow{V_n}$;

$\overrightarrow{T_{S,n}}$: the direction at the satellite within the vertical plane in S_n containing S, orthogonal to $\overrightarrow{S_nS}$ and to $\overrightarrow{L_{S,n}}$, and oriented towards the North;

i: the inclination angle between the antenna azimuth axis (Az-axis) $\overrightarrow{u_{Az,S}}$ and the left hand side direction $\overrightarrow{L_{S,n}}$ of the satellite for station #n;

i_{Eq}: the inclination angle between the $\overrightarrow{u_{Az,S}}$ and $\overrightarrow{L_{S,n}}$ with the antenna mount.

Then:

$$\overrightarrow{u_{Az,S}} \quad \frac{\overrightarrow{N} \wedge \overrightarrow{S_nS}}{\left\| \overrightarrow{N} \wedge \overrightarrow{S_nS} \right\|} \tag{82}$$

$$\overrightarrow{u_{El,S}} = \frac{\overrightarrow{S_nS} \wedge \overrightarrow{u_{Az,S}}}{\left\| \overrightarrow{S_nS} \right\|} \tag{83}$$

$$i_{Eq} = angle\left(\overrightarrow{u_{Az,S}}, \overrightarrow{L_{S,n}}\right) \tag{84}$$

The values of $\sin(i)$ may be obtained from the following set of equations:

$$\overrightarrow{S_nS} = \begin{cases} x_S = \rho.\left(\sin(Lt_n).\cos(Lt_S).\cos(Lg_S - Lg_n) - \sin(Lt_S).\cos(Lt_n)\right) \\ y_S = \rho.\cos(Lt_S).\sin(Lg_S - Lg_n) \\ z_S = \rho.\left(\cos(Lt_n).\cos(Lt_S).\cos(Lg_S - Lg_n) + \sin(Lt_S).\sin(Lt_n)\right) - R \end{cases} \tag{85}$$

$$\overrightarrow{V_n} = \begin{cases} x = 0 \\ y = 0 \\ z = 1 \end{cases} \tag{86}$$

$$\overrightarrow{L_{S,n}} = \left(\frac{\overrightarrow{V_n} \wedge \overrightarrow{S_nS}}{\left\| \overrightarrow{V_n} \wedge \overrightarrow{S_nS} \right\|} \right) \tag{87}$$

$$\overrightarrow{T_{S,n}} = \left(\frac{\overrightarrow{S_nS}}{\left\| \overrightarrow{S_nS} \right\|} \right) \wedge \overrightarrow{L_{S,n}} \tag{88}$$

$$\overrightarrow{N} = \begin{cases} x = -\cos(Lt_n) \\ y = 0 \\ z = +\sin(Lt_n) \end{cases} \tag{89}$$

$$\overrightarrow{u_{Az,S}} \quad \frac{\overrightarrow{N} \wedge \overrightarrow{S_nS}}{\left\| \overrightarrow{N} \wedge \overrightarrow{S_nS} \right\|} \tag{90}$$

$$\begin{cases} \cos\left(i_{Eq}\right) = \overrightarrow{u_{Az,S}}.\overrightarrow{L_{S,n}} \\ \sin\left(i_{Eq}\right) = \overrightarrow{u_{Az,S}}.\overrightarrow{T_{S,n}} \end{cases} \Rightarrow \left\{ i_{Eq} = ArcTan2\left(\sin\left(i_{Eq}\right), \cos\left(i_{Eq}\right)\right) \right. \tag{91}$$

When the equatorial antenna mount is correctly installed, the antenna azimuth axis (Az-axis) inclination is: $i = -i_{Eq}$.

7 Axis alignment offsets and errors

7.1 Axis alignment errors with an azimuth-elevation mount

7.1.1 General

Let:

- S_n : the earth station with its antenna;
- $\overrightarrow{V_n}$: the vertical at station S_n;
- $\overrightarrow{\hat{V}_n}$: the estimated vertical at station S_n;
- S: the satellite;
- $\overrightarrow{L_{S,n}}$: the left hand side direction at the satellite for station S_n, orthogonal to $\overrightarrow{S_nS}$ and $\overrightarrow{V_n}$;
- $\overrightarrow{\hat{L}_{S,n}}$: the estimated left hand side direction at the satellite for station S_n, orthogonal to $\overrightarrow{S_nS}$ and $\overrightarrow{V_n}$;
- $\overrightarrow{N}$: the direction of the North;
- $\overrightarrow{\hat{N}}$: the estimated direction of the North at station S_n;
- $\overrightarrow{u_{Az,S}}$: the antenna azimuth axis (Az-axis) when the correct direction $\overrightarrow{N}$ of the North is used;
- $\overrightarrow{\hat{u}_{Az,S}}$: the antenna azimuth axis (Az-axis) when the estimated direction $\overrightarrow{\hat{N}}$ of the North is used;
- $\overrightarrow{T_{S,n}}$: the direction towards the antenna top: within the vertical plane $\left(\overrightarrow{V_n}, \overrightarrow{S_nS}\right)$ in S_n containing S, and orthogonal to $\overrightarrow{S_nS}$;
- $\overrightarrow{\hat{T}_{S,n}}$: the direction towards the antenna top: within the vertical plane $\left(\overrightarrow{\hat{V}_n}, \overrightarrow{S_nS}\right)$ in S_n containing S, and orthogonal to $\overrightarrow{S_nS}$;

i: the inclination angle between the antenna azimuth axis (Az-axis) $\overrightarrow{u_{Az,S}}$ and the left hand side direction $\overrightarrow{L_{S,n}}$ of the satellite for station #n when the correct direction $\overrightarrow{N}$ of the North is used;

(i+δi): the inclination angle between the antenna azimuth axis (Az-axis) $\overrightarrow{\hat{u}_{Az,S}}$ and the left hand side direction $\overrightarrow{L_{S,n}}$ of the satellite for station #n when the estimated direction $\overrightarrow{\hat{N}}$ of the North is used.

Then:

$$\overrightarrow{L_{S,n}} = \frac{\overrightarrow{V_n} \wedge \overrightarrow{S_nS}}{\left\| \overrightarrow{V_n} \wedge \overrightarrow{S_nS} \right\|} \quad (92)$$

$$\overrightarrow{\hat{L}_{S,n}} = \frac{\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}}{\left\| \overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS} \right\|} \quad (93)$$

$$\overrightarrow{T_{S,n}} = \left(\frac{\overrightarrow{S_nS}}{\left\| \overrightarrow{S_nS} \right\|} \right) \wedge \overrightarrow{L_{S,n}} \quad (94)$$

$$\overrightarrow{\hat{T}_{S,n}} = \left(\frac{\overrightarrow{S_nS}}{\left\| \overrightarrow{S_nS} \right\|} \right) \wedge \overrightarrow{\hat{L}_{S,n}} \quad (95)$$

$$\overrightarrow{u_{Az,S}} = \cos(i).\overrightarrow{L_{S,n}} + \sin(i).\overrightarrow{T_{S,n}} \quad (96)$$

$$\overrightarrow{\hat{u}_{Az,S}} = \cos(i).\overrightarrow{\hat{L}_{S,n}} + \sin(i).\overrightarrow{\hat{T}_{S,n}} \quad (97)$$

$$\overrightarrow{\hat{u}_{Az,S}} = \cos(i+\delta i).\overrightarrow{L_{S,n}} + \sin(i+\delta i).\overrightarrow{T_{S,n}} \quad (98)$$

$$\boxed{\overrightarrow{\hat{u}_{Az,S}}.\overrightarrow{u_{Az,S}} = \cos(\delta i)} \quad (99)$$

$$\overrightarrow{\hat{u}_{Az,S}} \wedge \overrightarrow{u_{Az,S}} = \sin(\delta i).\overrightarrow{T_{S,n}} \wedge \overrightarrow{L_{S,n}} = \sin(\delta i).\frac{\overrightarrow{S_nS}}{\left\| \overrightarrow{S_nS} \right\|} \quad (100)$$

$$\boxed{\sin(\delta i) = \left(\overrightarrow{\hat{u}_{Az,S}} \wedge \overrightarrow{u_{Az,S}} \right).\frac{\overrightarrow{S_nS}}{\left\| \overrightarrow{S_nS} \right\|}} \quad (101)$$

7.1.2 Case of no alignment of the antenna azimuth axis with i = 0

In that clause the equivalent antenna azimuth axis inclination variation δi due to the use of an approximated vertical direction is determined for the case where the antenna mount antenna axis offset is equal to zero: $\boxed{i = 0}$.

$$\overrightarrow{u_{Az,S}} = \overrightarrow{L_{S,n}} \quad (102)$$

$$\overrightarrow{\hat{u}_{Az,S}} = \overrightarrow{\hat{L}_{S,n}} \quad (103)$$

$$\overrightarrow{\hat{u}_{Az,S}} = \cos(\delta i).\overrightarrow{L_{S,n}} + \sin(\delta i).\overrightarrow{T_{S,n}} \qquad (104)$$

$$\left(\overrightarrow{\hat{u}_{Az,S}} \wedge \overrightarrow{u_{Az,S}}\right).\overrightarrow{S_nS} = \left(\overrightarrow{\hat{L}_{S,n}} \wedge \overrightarrow{L_{S,n}}\right).\overrightarrow{S_nS} = \left(\left(\frac{\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}}{\left\|\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right\|}\right) \wedge \left(\frac{\overrightarrow{V_n} \wedge \overrightarrow{S_nS}}{\left\|\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right\|}\right)\right).\overrightarrow{S_nS} \qquad (105)$$

$$\begin{cases} a \quad +\left(\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right) \wedge \left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right)\right).\overrightarrow{S_nS} \\ a = +\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right).\left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \\ a = +\overrightarrow{\hat{V}_n}.\left(\overrightarrow{S_nS} \wedge \left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right)\right) \\ a = -\overrightarrow{\hat{V}_n}.\left(\left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \\ a = +\left\|\overrightarrow{S_nS}\right\|^2.\overrightarrow{\hat{V}_n}.\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \end{cases} \qquad \begin{cases} a = +\left\|\overrightarrow{S_nS}\right\|^2.\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right).\overrightarrow{\hat{V}_n} \\ a = +\left\|\overrightarrow{S_nS}\right\|^2.\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{V_n}\right).\overrightarrow{S_nS} \\ a = +\left\|\overrightarrow{S_nS}\right\|^2.\left(\overrightarrow{S_nS} \wedge \overrightarrow{\hat{V}_n}\right).\overrightarrow{V_n} \end{cases} \qquad (106)$$

$$\boxed{\sin(\delta i) = \left(\overrightarrow{\hat{u}_{Az,S}} \wedge \overrightarrow{u_{Az,S}}\right).\frac{\overrightarrow{S_nS}}{\left\|\overrightarrow{S_nS}\right\|} = \frac{\left\|\overrightarrow{S_nS}\right\|}{\left\|\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right\|}.\left(\frac{\left(\overrightarrow{S_nS} \wedge \overrightarrow{\hat{V}_n}\right)}{\left\|\overrightarrow{S_nS} \wedge \overrightarrow{\hat{V}_n}\right\|}\right).\overrightarrow{V_n}} \qquad (107)$$

The value of $\dfrac{\left\|\overrightarrow{S_nS}\right\|}{\left\|\overrightarrow{S_nS} \wedge \overrightarrow{V_n}\right\|}$ may be computed as follows:

$$\overrightarrow{V_n} = \begin{cases} x = 0 \\ y = 0 \\ z = 1 \end{cases} \qquad (108)$$

$$d_n \quad \left\|\overrightarrow{S_nS}\right\| \qquad (109)$$

$$\overrightarrow{S_nS} = \begin{cases} x = -d_n.\cos(El_S).\cos(Az_S) \\ y = +d_n.\cos(El_S).\sin(Az_S) \\ z = +d_n.\sin(El_S) \end{cases} \qquad (110)$$

$$\overrightarrow{S_nS} \wedge \overrightarrow{V_n} = \begin{cases} x = d_n.\cos(El_S).\sin(Az_S) \\ y = d_n.\cos(El_S).\cos(Az_S) \\ z = 0 \end{cases} \qquad (111)$$

$$\left\|\overrightarrow{S_nS} \wedge \overrightarrow{V_n}\right\| = d_n.\cos(El_S) \qquad (112)$$

$$\frac{\left\|\overrightarrow{S_nS}\right\|}{\left\|\overrightarrow{S_nS} \wedge \overrightarrow{V_n}\right\|} = \frac{1}{\cos(El_S)} \qquad (113)$$

Then:

$$\sin(\delta i) = \frac{1}{\cos(El_S)} \cdot \left(\frac{\left(\overrightarrow{S_nS} \wedge \overrightarrow{\hat{V}_n}\right)}{\left\| \overrightarrow{S_nS} \wedge \overrightarrow{\hat{V}_n} \right\|} \right) . \overrightarrow{V_n} \tag{114}$$

The computation of the maximum value of $\sin(\delta i)$ for given values of the satellite elevation El_S and of the vertical offset $\left\| \overrightarrow{V_n} \wedge \overrightarrow{\hat{V}_n} \right\|$ are simplified when using a local Cartesian coordinate system $(S_n, \tilde{x}, \tilde{y}, \tilde{z})$ oriented such that the satellite is within the vertical plane $(\tilde{x}, \tilde{z})$, as represented on figure 47:

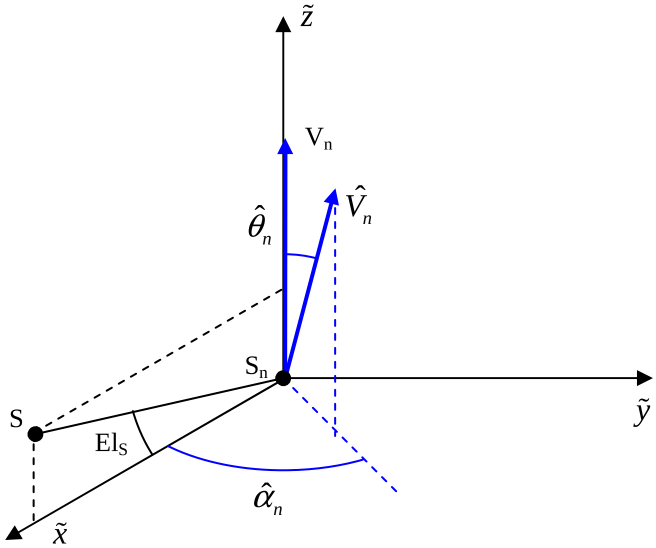

Figure 47: Local Cartesian coordinate system $(S_n, \tilde{x}, \tilde{y}, \tilde{z})$

Let:

$(S_n, \tilde{x}, \tilde{y}, \tilde{z})$: the Cartesian coordinate system where, $\hat{z}$ is the vertical in S_n, $(\tilde{x}, \tilde{y})$ is the horizontal plane in S_n, $(\tilde{x}, \tilde{z})$ is the vertical plane containing the satellite S;

$\hat{\theta}$: the offset angle of the estimated vertical $\overrightarrow{\hat{V}_n}$ relative to the vertical $\overrightarrow{V_n}$: $\left\| \overrightarrow{V_n} \wedge \overrightarrow{\hat{V}_n} \right\| \quad \sin(\hat{\theta})$;

$\hat{\alpha}$: the angle between the projection of $\overrightarrow{\hat{V}_n}$ in the horizontal plane with the $\tilde{x}$ axis.

Then:

$$\overrightarrow{V_n} = \begin{cases} \tilde{x} = 0 \\ \tilde{y} = 0 \\ \tilde{z} = 1 \end{cases} \tag{115}$$

$$\frac{\overrightarrow{S_nS}}{\left\|\overrightarrow{S_nS}\right\|} = \begin{cases} \tilde{x} = \cos(El_S) \\ \tilde{y} = 0 \\ \tilde{z} = \sin(El_S) \end{cases} \tag{116}$$

$$\vec{\hat{V}}_n = \begin{cases} \tilde{x} = \sin(\hat{\theta}).\cos(\hat{\alpha}) \\ \tilde{y} = \sin(\hat{\theta}).\sin(\hat{\alpha}) \\ \tilde{z} = \cos(\hat{\theta}) \end{cases} \tag{117}$$

$$\frac{\overrightarrow{S_nS}}{\left\|\overrightarrow{S_nS}\right\|} \wedge \vec{\hat{V}}_n = \begin{cases} \tilde{x} = -\sin(El_S).\sin(\hat{\theta}).\sin(\hat{\alpha}) \\ \tilde{y} = +\sin(El_S).\sin(\hat{\theta}).\cos(\hat{\alpha}) - \cos(El_S).\cos(\hat{\theta}) \\ \tilde{z} = +\cos(El_S).\sin(\hat{\theta}).\sin(\hat{\alpha}) \end{cases} \tag{118}$$

$$\left(\frac{\overrightarrow{S_nS}}{\left\|\overrightarrow{S_nS}\right\|} \wedge \vec{\hat{V}}_n\right)\overrightarrow{V_n} = \cos(El_S).\sin(\hat{\theta}).\sin(\hat{\alpha}) \tag{119}$$

$$\left\|\frac{\overrightarrow{S_nS}}{\left\|\overrightarrow{S_nS}\right\|} \wedge \vec{\hat{V}}_n\right\|^2 = \left(\sin(\hat{\theta}).\sin(\hat{\alpha})\right)^2 + \left(\sin(El_S).\sin(\hat{\theta}).\cos(\hat{\alpha}) - \cos(El_S).\cos(\hat{\theta})\right)^2 \tag{120}$$

$$\sin(\delta i) = \frac{\sin(\hat{\theta}).\sin(\hat{\alpha})}{\sqrt{\left(\sin(\hat{\theta}).\sin(\hat{\alpha})\right)^2 + \left(\sin(El_S).\sin(\hat{\theta}).\cos(\hat{\alpha}) - \cos(El_S).\cos(\hat{\theta})\right)^2}} \tag{121}$$

$$\sin(\delta i)^2 = \frac{\left(\sin(\hat{\theta}).\sin(\hat{\alpha})\right)^2}{\left(\sin(\hat{\theta}).\sin(\hat{\alpha})\right)^2 + \left(\sin(El_S).\sin(\hat{\theta}).\cos(\hat{\alpha}) - \cos(El_S).\cos(\hat{\theta})\right)^2} \tag{122}$$

The maximum values, as the minimum values, of δi for given values of El_S and $\hat{\theta}$ are obtained for values of $\hat{\alpha}$ which are solutions of:

$$\frac{d\left(\sin(\delta i)^2\right)}{d\hat{\alpha}} = 0 \tag{123}$$

with:

$$\frac{d\left(\sin(\delta i)^2\right)}{d\hat{\alpha}} = \frac{-2.\sin(\hat{\theta})^2.\sin(\hat{\alpha}).\begin{pmatrix} \left(\cos(El_S).\cos(\hat{\theta}) - \sin(El_S).\sin(\hat{\theta}).\cos(\hat{\alpha})\right). \\ \left(\sin(El_S).\sin(\hat{\theta}) - \cos(El_S).\cos(\hat{\theta}).\cos(\hat{\alpha})\right) \end{pmatrix}}{\left(\left(\sin(\hat{\theta}).\sin(\hat{\alpha})\right)^2 + \left(\sin(El_S).\sin(\hat{\theta}).\cos(\hat{\alpha}) - \cos(El_S).\cos(\hat{\theta})\right)^2\right)^2} \tag{124}$$

There are 3 values of $\hat{\alpha}$ which make null the derivative:

$$\frac{d\left(\sin\left(\delta i\right)^2\right)}{d\hat{\alpha}} = 0 \qquad \Leftrightarrow \begin{cases} \hat{\alpha} = k.\pi \\ \cos\left(\hat{\alpha}\right) = tg\left(El_S\right).tg\left(\hat{\theta}\right) \\ \cos\left(\hat{\alpha}\right) = ctg\left(El_S\right).ctg\left(\hat{\theta}\right) \end{cases} \qquad (125)$$

The first solution corresponds to the minimum value of δi :

$$\hat{\alpha} = k.\pi \qquad \Rightarrow \qquad \sin\left(\delta i\right)^2 = \frac{0}{0 + \cos\left(El_S + \hat{\theta}\right)^2} = 0 \qquad for\ El_S \neq \frac{\pi}{2} - \hat{\theta} \qquad (126)$$

The second solution corresponds to the maximum value of δi for $0 \leq \left|El_S\right| \leq \frac{\pi}{2} - \left|\hat{\theta}\right|$:

$$\cos\left(\hat{\alpha}\right) = tg\left(El_S\right).tg\left(\hat{\theta}\right) \qquad \Rightarrow \left|\sin\left(\delta i\right)\right| = \frac{\sin\left(\left|\hat{\theta}\right|\right)}{\cos\left(\left|El_S\right|\right)} \qquad (127)$$

$$\left|tg\left(El_S\right).tg\left(\hat{\theta}\right)\right| \leq 1 \qquad \Leftrightarrow \qquad ctg\left(\left|El_S\right|\right) \geq tg\left(\left|\hat{\theta}\right|\right) \qquad \Leftrightarrow \qquad 0 \leq \left|El_S\right| \leq \frac{\pi}{2} - \left|\hat{\theta}\right| \qquad (128)$$

The third solution corresponds to the maximum value of δi for $\frac{\pi}{2} - \left|\hat{\theta}\right| \leq \left|El_S\right| \leq \frac{\pi}{2}$:

$$\cos\left(\hat{\alpha}\right) = ctg\left(El_S\right).ctg\left(\hat{\theta}\right) \qquad \Rightarrow \qquad \left|\sin\left(\delta i\right)\right| = 1 \qquad (129)$$

$$\left|ctg\left(El_S\right).ctg\left(\hat{\theta}\right)\right| \leq 1 \quad \Leftrightarrow \qquad ctg\left(\left|El_S\right|\right) \leq tg\left(\left|\hat{\theta}\right|\right) \qquad \Leftrightarrow \qquad \frac{\pi}{2} - \left|\hat{\theta}\right| \leq \left|El_S\right| \leq \frac{\pi}{2} \qquad (130)$$

The conclusion is the following:

- For a antenna with an azimuth-elevation mount, designed for a maximum elevation $El_{S,\max}$ and for a maximum vertical axis offset $\hat{\theta}_{\max}$ then the maximum value $\delta i_{\max}$ of the offset δi of the antenna azimuth axis $\overrightarrow{u_{Az,S}}$ is given by the following equations:

  - for $0 \leq \left|El_{S,\max}\right| \leq \frac{\pi}{2} - \left|\hat{\theta}_{\max}\right|$: $\qquad \delta i_{\max} = ArcSin\left(\frac{\sin\left(\left|\hat{\theta}_{\max}\right|\right)}{\cos\left(\left|El_{S,\max}\right|\right)}\right)$ (130a)

  - for $\frac{\pi}{2} - \left|\hat{\theta}_{\max}\right| \leq \left|El_{S,\max}\right| \leq \frac{\pi}{2}$: $\qquad \delta i_{\max} = \frac{\pi}{2} rad$ (130b)

At given latitude Lt_n the maximum elevation of an antenna is for a satellite within the meridian plane of the antenna.

$$\cos\left(El_{\max}\right) = \frac{\rho.\sin\left(Lt_n\right)}{\sqrt{\rho^2 + R^2 - 2.\rho.R.\cos\left(Lt_n\right)}} \qquad (131)$$

The minimum is the latitude, the maximum is the elevation.

7.1.3 Case of no alignment of the antenna azimuth axis with i ≠ 0

In that clause the equivalent antenna azimuth axis inclination variation δi due to the use of an approximated vertical direction is determined for the case where the antenna mount antenna azimuth axis offset is not equal to zero: $\boxed{i \neq 0}$.

As for $i = 0$:

$$\overrightarrow{u_{Az,S}} = \cos(i).\overrightarrow{L_{S,n}} + \sin(i).\overrightarrow{T_{S,n}} \tag{132}$$

$$\overrightarrow{\hat{u}_{Az,S}} = \cos(i).\overrightarrow{\hat{L}_{S,n}} + \sin(i).\overrightarrow{\hat{T}_{S,n}} \tag{133}$$

$$\overrightarrow{\hat{u}_{Az,S}} = \cos(i+\delta i).\overrightarrow{L_{S,n}} + \sin(i+\delta i).\overrightarrow{T_{S,n}} \tag{134}$$

$$\boxed{\overrightarrow{\hat{u}_{Az,S}}.\overrightarrow{u_{Az,S}} = \cos(\delta i)} \tag{135}$$

$$\overrightarrow{\hat{u}_{Az,S}} \wedge \overrightarrow{u_{Az,S}} = \sin(\delta i).\overrightarrow{T_{S,n}} \wedge \overrightarrow{L_{S,n}} = \sin(\delta i).\frac{\overrightarrow{S_nS}}{\left\|\overrightarrow{S_nS}\right\|} \tag{136}$$

$$\boxed{\sin(\delta i) = \left(\overrightarrow{\hat{u}_{Az,S}} \wedge \overrightarrow{u_{Az,S}}\right).\frac{\overrightarrow{S_nS}}{\left\|\overrightarrow{S_nS}\right\|}} \tag{137}$$

but:

$$\sin(\delta i) = \frac{1}{\left\|\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right\|.\left\|\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right\|.\left\|\overrightarrow{S_nS}\right\|}.\begin{pmatrix} \cos(i)^2.\overrightarrow{V_{cc}} + \sin(i)^2.\dfrac{1}{\left\|\overrightarrow{S_nS}\right\|^2}.\overrightarrow{V_{ss}} \\ +\cos(i).\sin(i).\dfrac{1}{\left\|\overrightarrow{S_nS}\right\|}.\left(\overrightarrow{V_{sc}} + \overrightarrow{V_{cs}}\right) \end{pmatrix} \tag{138}$$

with:

$$\begin{cases} \overrightarrow{V_{cc}} & \left(\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right) \wedge \left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right)\right).\overrightarrow{S_nS} \\ \overrightarrow{V_{ss}} & \left(\left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right)\right) \wedge \left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right)\right)\right).\overrightarrow{S_nS} \\ \overrightarrow{V_{sc}} & \left(\left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right)\right) \wedge \left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right)\right).\overrightarrow{S_nS} \\ \overrightarrow{V_{cs}} & \left(\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right) \wedge \left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right)\right)\right).\overrightarrow{S_nS} \end{cases} \tag{139}$$

$$
\begin{cases}
\overrightarrow{V_{cc}} = \left(\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right) \wedge \left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right)\right).\overrightarrow{S_nS} \\
\overrightarrow{V_{cc}} = \left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right).\left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \\
\overrightarrow{V_{cc}} = \overrightarrow{\hat{V}_n}.\left(\overrightarrow{S_nS} \wedge \left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right)\right) \\
\overrightarrow{V_{cc}} = -\left(\left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right).\overrightarrow{\hat{V}_n} \\
\overrightarrow{V_{cc}} = +\left\|\overrightarrow{S_nS}\right\|^2.\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right).\overrightarrow{\hat{V}_n}
\end{cases} \tag{140}
$$

$$
\begin{cases}
\overrightarrow{V_{ss}} = \left(\left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right)\right) \wedge \left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right)\right)\right).\overrightarrow{S_nS} \\
\overrightarrow{V_{ss}} = \left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right)\right).\left(\left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right)\right) \wedge \overrightarrow{S_nS}\right) \\
\overrightarrow{V_{ss}} = \left(\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right).\left(\left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \\
\overrightarrow{V_{ss}} = -\left\|\overrightarrow{S_nS}\right\|^2.\left(\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right).\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right)
\end{cases} \tag{141}
$$

$$
\begin{cases}
\overrightarrow{V_{ss}} = -\left\|\overrightarrow{S_nS}\right\|^2.\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right).\left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right)\right) \\
\overrightarrow{V_{ss}} = +\left\|\overrightarrow{S_nS}\right\|^2.\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right).\left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \\
\overrightarrow{V_{ss}} = +\left\|\overrightarrow{S_nS}\right\|^2.\overrightarrow{\hat{V}_n}.\left(\overrightarrow{S_nS} \wedge \left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right)\right) \\
\overrightarrow{V_{ss}} = -\left\|\overrightarrow{S_nS}\right\|^2.\overrightarrow{\hat{V}_n}.\left(\left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \\
\overrightarrow{V_{ss}} = +\left\|\overrightarrow{S_nS}\right\|^4.\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right).\overrightarrow{\hat{V}_n}
\end{cases} \tag{142}
$$

$$
\begin{cases}
\overrightarrow{V_{sc}} = \left(\left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right)\right) \wedge \left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right)\right).\overrightarrow{S_nS} \\
\overrightarrow{V_{sc}} = \left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right)\right).\left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \\
\overrightarrow{V_{sc}} = \left(\left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right).\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right) \\
\overrightarrow{V_{sc}} = \left\|\overrightarrow{S_nS}\right\|^2.\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right).\left(\overrightarrow{S_nS} \wedge \overrightarrow{\hat{V}_n}\right) \\
\overrightarrow{V_{sc}} = \left\|\overrightarrow{S_nS}\right\|^2.\left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right).\overrightarrow{\hat{V}_n}
\end{cases} \tag{143}
$$

$$\begin{cases} \overrightarrow{V_{cs}} = \left(\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right) \wedge \left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right)\right)\right).\overrightarrow{S_nS} \\ \overrightarrow{V_{cs}} = -\left(\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right) \wedge \left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right)\right).\overrightarrow{S_nS} \\ \overrightarrow{V_{cs}} = -\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right).\left(\left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right) \\ \overrightarrow{V_{cs}} = \left\|\overrightarrow{S_nS}\right\|^2.\left(\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right).\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \\ \overrightarrow{V_{cs}} = \left\|\overrightarrow{S_nS}\right\|^2.\overrightarrow{\hat{V}_n}.\left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right)\right) \\ \overrightarrow{V_{cs}} = -\left\|\overrightarrow{S_nS}\right\|^2.\left(\left(\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right) \wedge \overrightarrow{S_nS}\right).\overrightarrow{\hat{V}_n} \end{cases} \qquad (144)$$

and finally:

$$\boxed{\sin(\delta i) = \frac{\left\|\overrightarrow{S_nS}\right\|}{\left\|\overrightarrow{\hat{V}_n} \wedge \overrightarrow{S_nS}\right\|}.\left(\frac{\overrightarrow{V_n} \wedge \overrightarrow{S_nS}}{\left\|\overrightarrow{V_n} \wedge \overrightarrow{S_nS}\right\|}\right).\overrightarrow{\hat{V}_n} = \frac{1}{\cos(El_S)}.\left(\frac{\left(\overrightarrow{S_nS} \wedge \overrightarrow{\hat{V}_n}\right)}{\left\|\overrightarrow{S_nS} \wedge \overrightarrow{\hat{V}_n}\right\|}\right).\overrightarrow{V_n}} \qquad (145)$$

This expression of $\sin(\delta i)$ is independent of i.

Consequently, the results obtained for $i = 0$ are also applicable for $i \neq 0$.

7.1.4 Case of alignment of the antenna azimuth axis

For an antenna mount with alignment possibility of making the antenna azimuth axis parallel to the tangent to the GSO at the satellite:

$$\overrightarrow{\hat{u}_{Az,S}} = \overrightarrow{u_{Az,S}} = \left(\frac{\overrightarrow{E_S} \wedge \overrightarrow{S_nS}}{\left\|\overrightarrow{E_S} \wedge \overrightarrow{S_nS}\right\|}\right) \wedge \overrightarrow{S_nS} \qquad and \qquad \overrightarrow{\hat{u}_{Az,S}} \wedge \overrightarrow{u_{Az,S}} = 0 \qquad (146)$$

then:

$$\sin(\delta i) = \left(\overrightarrow{\hat{u}_{Az,S}} \wedge \overrightarrow{u_{Az,S}}\right).\frac{\overrightarrow{S_nS}}{\left\|\overrightarrow{S_nS}\right\|} = 0 \qquad (147)$$

or:

$$\boxed{\delta i = 0} \qquad (148)$$

7.2 Axis alignment errors with an equatorial mount

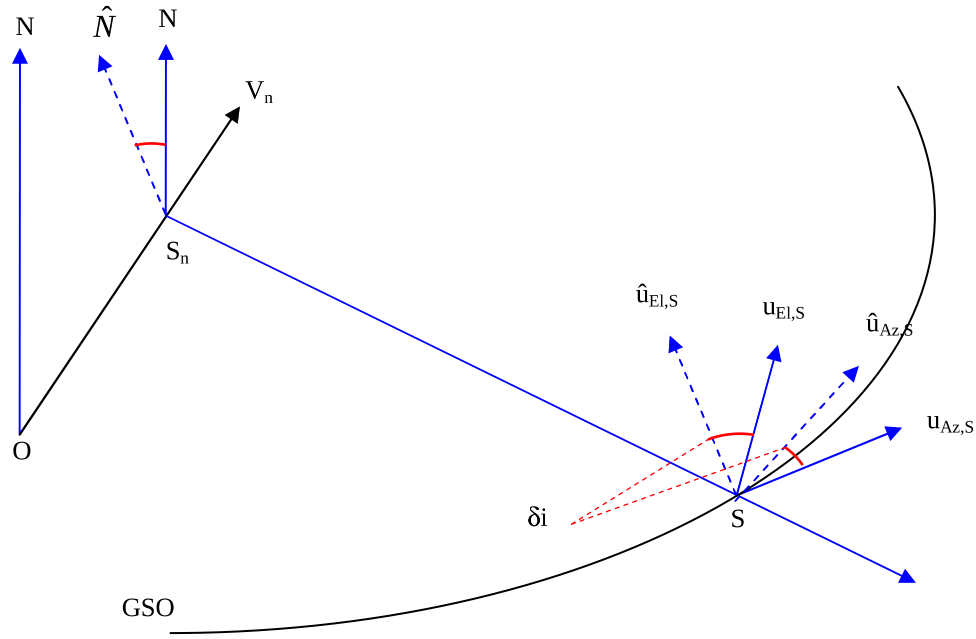

Figure 48: Estimated North direction $\overrightarrow{\hat{N}}$ and corresponding Az-axis $\overrightarrow{\hat{u}_{Az,S}}$

Let:

S_n: the earth station with its antenna,

$\overrightarrow{V_n}$: the vertical at station S_n,

S: the satellite;

$\overrightarrow{N}$: the direction of the North;

$\overrightarrow{\hat{N}}$: the estimated direction of the North at station S_n;

$\overrightarrow{u_{Az,S}}$: the antenna azimuth axis (Az-axis) when the correct direction $\overrightarrow{N}$ of the North is used;

$\overrightarrow{\hat{u}_{Az,S}}$: the antenna azimuth axis (Az-axis) when the estimated direction $\overrightarrow{\hat{N}}$ of the North is used;

$\overrightarrow{L_{S,n}}$: the left hand side direction at the satellite for station S_n: orthogonal to $\overrightarrow{S_nS}$ and $\overrightarrow{V_n}$;

$\overrightarrow{T_{S,n}}$: the direction at the satellite within the vertical plane in S_n containing S, orthogonal to $\overrightarrow{S_nS}$ and to $\overrightarrow{L_{S,n}}$, and oriented towards the antenna top;

i: the inclination angle between the antenna azimuth axis (Az-axis) $\overrightarrow{u_{Az,S}}$ and the left hand side direction $\overrightarrow{L_{S,n}}$ of the satellite for station #n when the correct direction $\overrightarrow{N}$ of the North is used;

(i+δi): the inclination angle between the antenna azimuth axis (Az-axis) $\overrightarrow{\hat{u}_{Az,S}}$ and the left hand side direction $\overrightarrow{L_{S,n}}$ of the satellite for station #n when the estimated direction $\overrightarrow{\hat{N}}$ of the North is used.

Then:

$$\overrightarrow{u_{Az,S}} = \frac{\overrightarrow{N} \wedge \overrightarrow{S_nS}}{\left\| \overrightarrow{N} \wedge \overrightarrow{S_nS} \right\|} \tag{149}$$

$$\overrightarrow{\hat{u}_{Az,S}} = \frac{\overrightarrow{\hat{N}} \wedge \overrightarrow{S_nS}}{\left\| \overrightarrow{\hat{N}} \wedge \overrightarrow{S_nS} \right\|} \tag{150}$$

$$\overrightarrow{u_{Az,S}} = \cos(i).\overrightarrow{L_{S,n}} + \sin(i).\overrightarrow{T_{S,n}} \tag{151}$$

$$\overrightarrow{\hat{u}_{Az,S}} = \cos(i+\delta i).\overrightarrow{L_{S,n}} + \sin(i+\delta i).\overrightarrow{T_{S,n}} \tag{152}$$

$$\overrightarrow{\hat{u}_{Az,S}}.\overrightarrow{u_{Az,S}} = \cos(\delta i) \tag{153}$$

$$\overrightarrow{\hat{u}_{Az,S}} \wedge \overrightarrow{u_{Az,S}} = \sin(\delta i).\overrightarrow{T_{S,n}} \wedge \overrightarrow{L_{S,n}} = \sin(\delta i).\frac{\overrightarrow{S_nS}}{\left\| \overrightarrow{S_nS} \right\|} \tag{154}$$

$$\sin(\delta i) = \left(\overrightarrow{\hat{u}_{Az,S}} \wedge \overrightarrow{u_{Az,S}} \right).\frac{\overrightarrow{S_nS}}{\left\| \overrightarrow{S_nS} \right\|} \tag{155}$$

$$\sin(\delta i) = \left(\left(\frac{\overrightarrow{\hat{N}} \wedge \overrightarrow{S_nS}}{\left\| \overrightarrow{\hat{N}} \wedge \overrightarrow{S_nS} \right\|} \right) \wedge \left(\frac{\overrightarrow{N} \wedge \overrightarrow{S_nS}}{\left\| \overrightarrow{N} \wedge \overrightarrow{S_nS} \right\|} \right) \right).\frac{\overrightarrow{S_nS}}{\left\| \overrightarrow{S_nS} \right\|} \tag{156}$$

$$\left(\left(\overrightarrow{\hat{N}} \wedge \overrightarrow{S_nS} \right) \wedge \left(\overrightarrow{N} \wedge \overrightarrow{S_nS} \right) \right).\overrightarrow{S_nS} =$$

$$\begin{cases} = \overrightarrow{S_nS}.\left(\left(\overrightarrow{\hat{N}} \wedge \overrightarrow{S_nS} \right) \wedge \left(\overrightarrow{N} \wedge \overrightarrow{S_nS} \right) \right) = \left(\overrightarrow{S_nS} \wedge \left(\overrightarrow{\hat{N}} \wedge \overrightarrow{S_nS} \right) \right).\left(\overrightarrow{N} \wedge \overrightarrow{S_nS} \right) \\ = \left(\left(\overrightarrow{\hat{N}} \wedge \overrightarrow{S_nS} \right) \wedge \overrightarrow{S_nS} \right).\left(\overrightarrow{S_nS} \wedge \overrightarrow{N} \right) = \left(\left(\left(\overrightarrow{\hat{N}} \wedge \overrightarrow{S_nS} \right) \wedge \overrightarrow{S_nS} \right) \wedge \overrightarrow{S_nS} \right).\overrightarrow{N} \\ = -\left\| \overrightarrow{S_nS} \right\|^2.\left(\overrightarrow{\hat{N}} \wedge \overrightarrow{S_nS} \right).\overrightarrow{N} = -\left\| \overrightarrow{S_nS} \right\|^2.\overrightarrow{N}.\left(\overrightarrow{\hat{N}} \wedge \overrightarrow{S_nS} \right) = -\left\| \overrightarrow{S_nS} \right\|^2.\left(\overrightarrow{N} \wedge \overrightarrow{\hat{N}} \right).\overrightarrow{S_nS} \\ = \left\| \overrightarrow{S_nS} \right\|^2.\left(\overrightarrow{\hat{N}} \wedge \overrightarrow{N} \right).\overrightarrow{S_nS} = \left\| \overrightarrow{S_nS} \right\|^2.\overrightarrow{\hat{N}}.\left(\overrightarrow{N} \wedge \overrightarrow{S_nS} \right) = -\left\| \overrightarrow{S_nS} \right\|^2.\left(\overrightarrow{S_nS} \wedge \overrightarrow{N} \right).\overrightarrow{\hat{N}} \end{cases} \tag{157}$$

$$\boxed{\sin(\delta i) = -\frac{\left\| \overrightarrow{S_nS} \right\|}{\left\| \overrightarrow{S_nS} \wedge \overrightarrow{\hat{N}} \right\|}.\left(\frac{\overrightarrow{S_nS} \wedge \overrightarrow{N}}{\left\| \overrightarrow{S_nS} \wedge \overrightarrow{N} \right\|} \right).\overrightarrow{\hat{N}}} \tag{158}$$

$$\boxed{\sin(\delta i) = -\frac{1}{\sin\left(\overrightarrow{\hat{N}}, \overrightarrow{S_nS} \right)}.\left(\frac{\overrightarrow{S_nS} \wedge \overrightarrow{N}}{\left\| \overrightarrow{S_nS} \wedge \overrightarrow{N} \right\|} \right).\overrightarrow{\hat{N}}} \tag{159}$$

$$\sin\left(\overrightarrow{\hat{N}}, \overrightarrow{S_n S}\right) \approx 1 \qquad \Rightarrow \qquad \sin(\delta i) \approx -\left(\frac{\overrightarrow{S_n S} \wedge \overrightarrow{N}}{\left\|\overrightarrow{S_n S} \wedge \overrightarrow{N}\right\|}\right).\overrightarrow{\hat{N}} \qquad (160)$$

For a given location S_n, a given satellite S and for a given inclination error δi the ends of the vector $\overrightarrow{\hat{N}}$ representing the estimated directions of the North at station S_n are on a circle in a plane parallel to the plane $\left(\overrightarrow{S_n S}, \overrightarrow{N}\right)$ and at a distance equal to $\sin(\delta i)$ from S_n on a sphere of radius equal to 1.

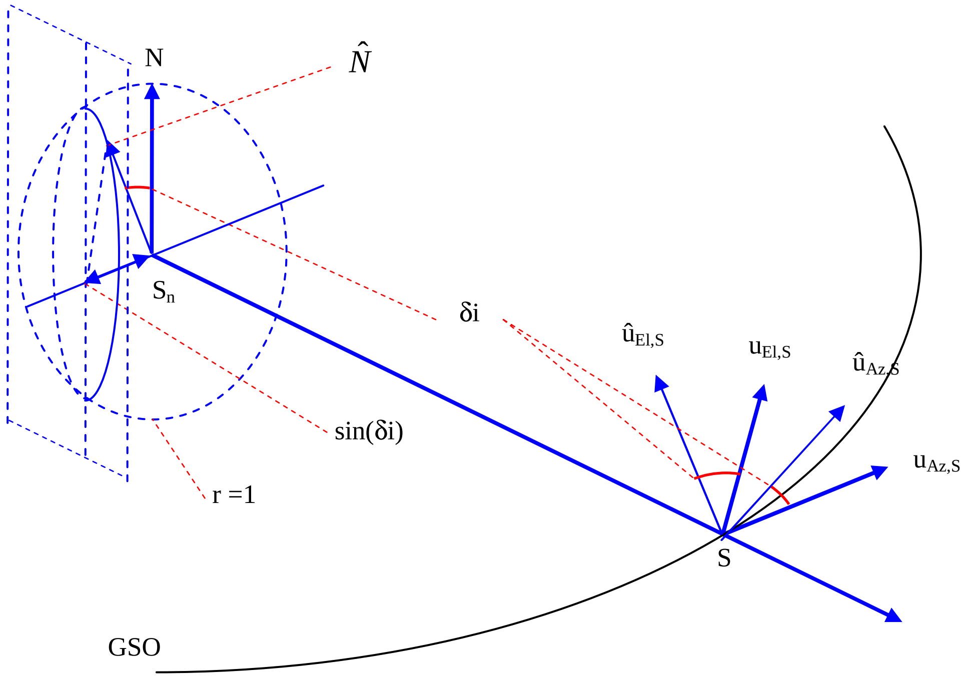

Figure 49: Circle of ends of vectors $\overrightarrow{\hat{N}}$ representing the estimated North directions for a given inclination error δi

For a given location S_n, any satellite S and for a given maximum inclination error Δi the ends of the vector $\overrightarrow{\hat{N}}$ representing the estimated directions of the North at station S_n are within the cone $\overrightarrow{N}.\overrightarrow{\hat{N}} = \cos(\Delta i)$ of axis $\overrightarrow{N}$ and on a sphere of radius equal to 1 centred in S_n.

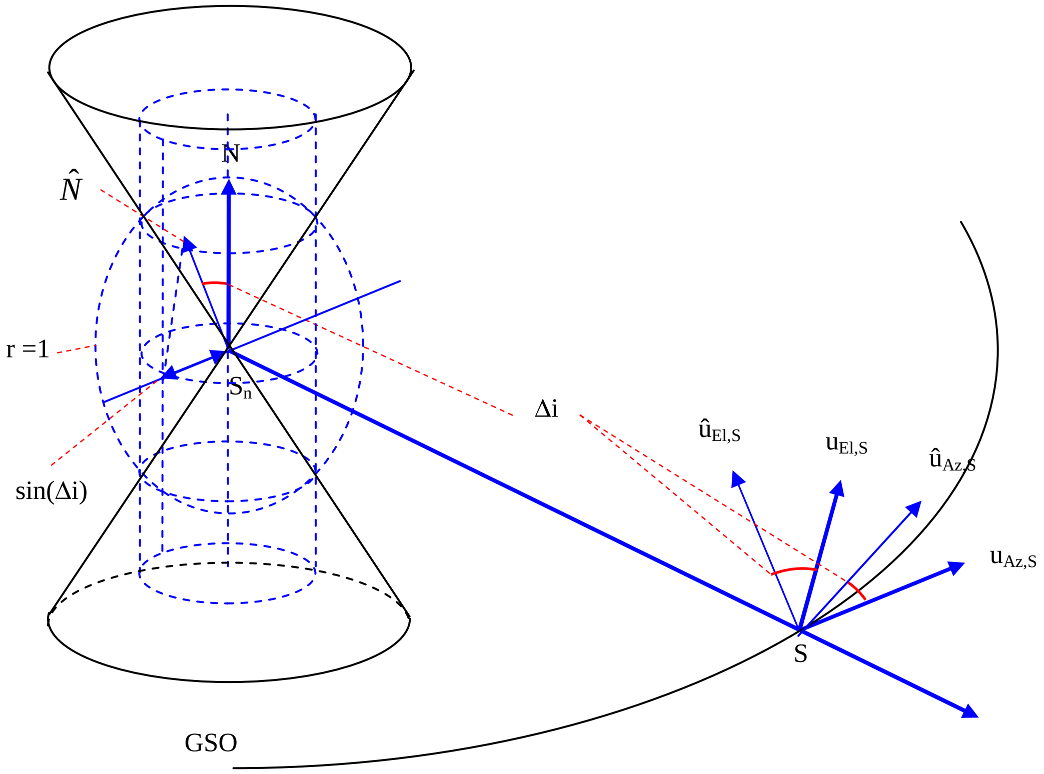

Figure 50: Cone of the vectors $\vec{\hat{N}}$ representing the estimated North directions for a given maximum inclination error Δi

Then the maximum inclination error Δi is also the maximum angle between the actual direction $\vec{N}$ of the North and the estimated direction $\vec{\hat{N}}$ of the North.

Let:

$El_{N,n}$:	the elevation of the direction $\vec{N}$ of the North pole in station S_n ;
$Az_{N,n}$:	the azimuth of the direction $\vec{N}$ of the North pole in station S_n ;
$\left(El_{N,n} + \delta El_{N,n}\right)$:	the elevation of the estimated direction $\vec{\hat{N}}$ of the North pole in station S_n ;
$\left(Az_{N,n} + \delta Az_{N,n}\right)$:	the azimuth of the estimated direction $\vec{\hat{N}}$ of the North pole in station S_n ;
ΔEl_N :	the maximum absolute value of $\delta El_{N,n}$: $\left\|\delta El_{N,n}\right\| \le \Delta El_N$;
ΔAz_N :	the maximum absolute value of $\delta Az_{N,n}$: $\left\|\delta Az_{N,n}\right\| \le \Delta Az_N$;
$Lt_{n,\min}$:	the maximum operational latitude for which the earth station is designed.

Then:

$$\overrightarrow{N} = \begin{cases} x = -\cos\left(Lt_n\right) & = -\cos\left(El_{N,n}\right).\cos\left(Az_{N,n}\right) \\ y = 0 & = +\cos\left(El_{N,n}\right).\sin\left(Az_{N,n}\right) \\ z = +\sin\left(Lt_n\right) & = +\sin\left(El_{N,n}\right) \end{cases} \quad (161)$$

$$\overrightarrow{\hat{N}} = \begin{cases} x = -\cos\left(El_{N,n} + \delta El_{N,n}\right).\cos\left(Az_{N,n} + \delta Az_{N,n}\right) \\ y = +\cos\left(El_{N,n} + \delta El_{N,n}\right).\sin\left(Az_{N,n} + \delta Az_{N,n}\right) \\ z = +\sin\left(El_{N,n} + \delta El_{N,n}\right) \end{cases} \quad (162)$$

and consequently:

$$\begin{cases} El_{N,n} = Lt_n \\ Az_{N,n} = 0 \end{cases} \quad (163)$$

$$\overrightarrow{N}.\overrightarrow{\hat{N}} = 1 - 2.\sin\left(\frac{\delta El_{N,n}}{2}\right)^2 - 2.\sin\left(\frac{\delta Az_{N,n}}{2}\right)^2.\cos\left(Lt_n\right).\cos\left(Lt_n + \delta El_{N,n}\right) \quad (164)$$

The following condition:

$$\overrightarrow{N}.\overrightarrow{\hat{N}} \geq \cos\left(\Delta i\right) = 1 - 2.\sin\left(\frac{\Delta i}{2}\right)^2 \quad (165)$$

becomes:

$$\sin\left(\frac{\delta El_{N,n}}{2}\right)^2 + \sin\left(\frac{\delta Az_{N,n}}{2}\right)^2.\cos\left(Lt_n\right).\cos\left(Lt_n + \delta El_{N,n}\right) \leq \sin\left(\frac{\Delta i}{2}\right)^2 \quad (166)$$

If ΔAz_N and ΔEl_N are the maximum absolute values of $\delta Az_{N,n}$ and $\delta El_{N,n}$, and if $Lt_{n,\min}$ is the maximum operational latitude for which the earth station is designed, then:

$$\boxed{\sin\left(\frac{\Delta El_{N,n}}{2}\right)^2 + \sin\left(\frac{\Delta Az_{N,n}}{2}\right)^2.\cos\left(Lt_{n,\min}\right).\cos\left(Lt_{n,\min} + \Delta El_{N,n}\right) \leq \sin\left(\frac{\Delta i}{2}\right)^2} \quad (167)$$

When the latitude of the station is known with an accuracy highly better than Δi, e.g. with a GPS, then the maximum error in azimuth of the direction of the local North or South is given by the following relationship:

$$\boxed{\Delta El_{N,n} = 0 \qquad \Rightarrow \quad \sin\left(\frac{\left|\Delta Az_{N,n}\right|}{2}\right) \leq \frac{\sin\left(\frac{\left|\Delta i\right|}{2}\right)}{\cos\left(Lt_n\right)}} \quad (168)$$

NOTE: A North-South error of 1 km on the location of the earth station corresponds to an error of 0,009° (= 360° x 1 km / 40 000 km) on the latitude of the earth station. This error is considered negligible for the antenna axis alignment. A GPS gives a better accuracy.

For antennas designed to operate at any latitude the condition becomes the following:

$$\left.\begin{cases}\Delta El_{N,n} = 0 \\ Lt_n \in \left[-80°, +80°\right]\end{cases}\right\} \Rightarrow \quad \left|\Delta Az_{N,n}\right| \leq \left|\Delta i\right| \tag{169}$$

8 Contour computation method

8.1 General

The computation of the contour of the shadows of the visible part of the GSO on an antenna radiation pattern is required for the following three cases:

a) the case of a station in a given location pointed towards a given satellite;

b) the case of stations within a range of latitudes pointed towards any satellites on the GSO and above a given elevation, including the case of a station in a given location pointed towards any satellites on the GSO and above a given elevation.

The functions referred to within the present clause 8 are described in detail in clause 9 (Excel Tool).

The method of the computation of the contour is base on the type of antenna mount considered within the present document. For any other type of antenna mount the same methodology shall be followed for the determination of the normal inclination of the antenna Az-axis and for the determination of the impact of the antenna mount axes alignment errors on the antenna Az-axis inclination and the methods described within the following two clauses 8.2 and 8.3 shall apply.

8.2 Case of 1 ES and 1 satellite

In the case of a station in a given location pointed towards a given satellite, for the determination of the shadow of the GSO and its vicinity on the antenna radiation pattern, within the (α, φ) domain, the method described in clause 9.7.2 shall apply.

The GSO shadow of the GSO and its vicinity shall be computed:

a) for no alignment error;

b) for the maximum positive alignment error;

c) for the maximum negative alignment error; and

d) if necessary when the maximum alignment error is large for intermediate alignment errors.

These computations shall be repeated for:

e) for the maximum inclination offset i of the antenna Az-axis;

f) for the minimum inclination offset i of the antenna Az-axis; and

g) if necessary when the range of inclination offset is large for intermediate values.

For antennas which could be used up-side down, all the above computations shall be repeated for the inclination offset increased or decreased by 180°.

The contour of the shadows of the GSO and its vicinity, with alignment errors and inclination offsets, on the antenna radiation pattern, within the (α, φ) domain, is the envelop of all the shadows which have been computed.

8.3 Case of ESs and satellites

In the case of stations within a range of latitudes pointed towards any satellites on the GSO and above a given elevation, including the case of a station in a given location pointed towards any satellites on the GSO and above a given elevation, for the determination of the contour of the shadows of the GSO and its vicinity on the antenna radiation patterns, within the (α, φ) domain, the method described in clause 9.6.13 shall apply.

The contour of the shadows of the GSO and its vicinity shall be computed:

a) for no alignment error;

b) for the maximum positive alignment error;

c) for the maximum negative alignment error; and

d) if necessary when the maximum alignment error is large for intermediate alignment errors.

These computations shall be repeated for:

e) for the maximum inclination offset i of the antenna Az-axis;

f) for the minimum inclination offset i of the antenna Az-axis; and

g) if necessary when the range of inclination offset is large for intermediate values.

For antennas which could be used up-side down, all the above computations shall be repeated for the inclination offset increased or decreased by 180°.

The contour of the shadows of the GSO and its vicinity, with alignment errors and inclination offsets, on the antenna radiation patterns, within the (α, φ) domain, is the envelop of all the contours which have been computed.

9 Excel Tool

9.1 General

It is recognized that all the problems of limits and discontinuities have not been fully considered, mainly when crossing the equator or in the vicinity of the sub-satellite point on the Earth. However it is considered that this tool provide enough information on the limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity for measurement purpose.

The Tool consists in:

- a worksheet, named "GSO";
- 2 Visual Basic Modules, named: "Basic_functions" and "GSO_arc_functions".

The "GSO" worksheet contains:

- 3 sections for: the constants, the input data, the GSO shadow output data, the contour output data;
- 4 graphs:
  - the GSO arc and its vicinity for an observer at the ES (the elevation vs. azimuth of the GSO points);
  - the GSO arc and its vicinity for the antenna (Phi_El vs. Phi_Az of the GSO points);
  - the GSO arc and its vicinity within the (α, φ) domain and the limit within the (α, φ) domain of the shadow of the GSO arc and its vicinity (Phi.sin(Alpha) vs. Phi.cos(Alpha) of the GSO points);
  - the minimum off-axis angle, corresponding to the minimum satellite longitude offset, within the (α, φ) domain.

9.2 Cell colours

Mainly 4 colours are used for the cells of the worksheet:

Brown for constants
Blank for input data
Blue for formulas or results from a Visual Basic subroutine
Green for titles

9.3 Constants

Presently the worksheet contains only one constant:

Constants		Remark
Antenna type table	1: Az-El mount without alignment	This table is use to display the type of antenna selected by a number.
	2: Az-El mount WITH ideal alignment	
	3: Az-El mount WITH polar alignment	
	4: Equatorial mount	

Other constants are declared:

Name	Value	Unit	Remark
Pi_	= 3,14159265358979		
dg	= Pi_/180	rad/°	
p_	= 42 164	km	GSO radius
R_	= 6 371	km	Earth radius
Sqr_2	= 1,4142135623731		$= \sqrt{2}$

9.4 Buttons

Two types of buttons are used within the worksheet:

- one "Command button" with caption "Contour" which initiate the computation of the contour with the input parameters;
- several "Spin buttons" () used to increment or decrement values of some input parameters, e.g.:

Title	Real variable	Integer variable	Spin button
ES latitude max	74,000°/N	74°/N	
ES latitude min	-74,000°/N	65 462°/N	

When the spin button value of a row is incremented or decremented, the associated integer variable within the row (e.g. i_ES_Lt_min) and the real variable (e.g. ES_Lt_min) of the row are updated with the spin button integer value. Any modification of the integer variable results in a modification of the spin button value with that value. A negative (e.g. -74°) value may be given to an integer variable but after action on the spin button the number displayed will be equal to $-74 + 2^{16}$.

There is an exception for "Lg_S0" where the integer variable is an integer between -100 and 100 and representing a percentage of the maximum longitude offset of the other satellite.

Title	Real variable	Integer variable	Spin button
Lg_S0	70,553°/E	100	

9.5 Input data

Excel worksheet input data area				Remarks
Inputs				
Antenna mount				
Type of antenna mount	2	2: Az-El mount WITH ideal alignment		Type of antenna: 1: Az-El mount without alignment 2: Az-El mount WITH alignment 3: Az-El mount WITH polar. alignment 4: equatorial mount
Permanent alignment error	0,000°/N	0		∈[-90°, +90°] The sign of this offset is given to the inclination error.
Az-El antenna mount				
Vertical direction offset	0,000°			Recommended values ∈[-180°, +180°] The sign of this offset is given to the inclination error.
Equatorial antenna mount				
North pole direction azimuth offset	4,000°			Recommended values ∈ [-0°, +360°]
North pole direction elevation offset	3,000°			Recommended values ∈ [-90°, +90°] The sign of this offset is given to the inclination error.
E/H field alignment				
Tilt angle with the North pole direction of the electric field at the satellite	22,000°			Recommended values ∈ [-180°, +180°]
Longitude of the centre of the satellite beam coverage	5,000°/E			Recommended values ∈ [-180°, +180°]
Latitude of the centre of the satellite beam coverage	50,000°/E			Recommended values ∈ [-90°, +90°]
H_polar	TRUE			TRUE for alignment on the received H field FALSE for alignment on the received E field
E/H field alignment error	0,000°			Recommended values ∈ [-180°, +180°] The sign of this offset is given to the inclination error.
Antenna inclination				
i_Az	-127,930°			Inclination with Az-El mount
i_Eq	52,700°			Inclination with equatorial mount
I_E	74,724°			Inclination with E or H field alignment
i_0	-127,930°			The selected antenna inclination according to the antenna type
Alignment errors				
Alignment error due to Az-El mount	0,000°			Computed with the axes alignment error
Alignment error due to Az-El mount with E field	0,000°			No formula available
Alignment error due to equatorial mount	4,654°			Computed with the axes alignment error
Alignment error	0,000°			Selected alignment error according to the antenna mount type
Earth station				
ES latitude	36,000°/N	36°/N		∈[-90°, +90°]
ES longitude	0,000°/E	0°/E		∈[-180°, +180°]
Minimum ES elevation	7,000°	7°		∈[-90°, +90°], of the pointed satellite
Minimum horizon elevation	0,000°	0°		∈ [-90°, +90°], of the other satellite
Max longitude offset	70,553°			Function of the minimum ES elevation
Pointed satellite				
Lg_S0	70,553°/E	100		= longitude of the ES + percentage of the maximum longitude offset, with percentage ∈[-100, +100]
Azimuth_0	101,724°			Azimuth of the pointed satellite
Elevation_0	7,000°			Elevation of the pointed satellite

Excel worksheet input data area				Remarks
Range of ES latitudes				
ES latitude min	-74,000°/N	65 462°/N	◂ ▸	
ES latitude max	74,000°/N	74°/N	◂ ▸	
ES latitude step	1,480°/N			Latitude step for the computations
Markers				
Phi	173,000°	173°	◂ ▸	Marker Phi = constant
Alpha	0,000°	0°	◂ ▸	Marker Alpha = constant

The minimum horizon elevation could be negative for an ES high above the see level:

ES altitude	Horizon elevation
0 m	-0,000°
100 m	-0,321°
200 m	-0,454°
500 m	-0,718°
1 000 m	-1,015°
1 500 m	-1,243°
2 000 m	-1,435°
3 000 m	-1,758°
4 000 m	-2,030°

9.6 Functions and Subroutines

The following Visual Basic functions and subroutines of the module "GSO_arc_functions" are used.

9.6.1 Function Azimuth()

The function Azimuth gives the satellite azimuth [°/N].

The inputs of the function Phi_ Azimuth(Lg_S, Lt_S, Lg_n, Lt_n) are:

- Lg_S the satellite longitude [°/E];
- Lt_S the satellite latitude [°/N];
- Lg_n the ES longitude [°/E];
- Lt_n the ES latitude [°/N].

The value of the function Azimuth is computed with the following equation:

$$Az_n = ArcTan2\left(\left(\sin\left(Lg_S - Lg_n\right)\right), \begin{pmatrix} -\sin\left(Lt_n\right).\cos\left(Lg_S - Lg_n\right) \\ +\tan\left(Lt_S\right).\cos\left(Lt_n\right) \end{pmatrix} \right) \tag{170}$$

9.6.2 Function Elevation()

The function Elevation gives the satellite elevation [°/E].

The inputs of the function Phi_ Elevation(Lg_S, Lt_S, Lg_n, Lt_n) are:

- Lg_S the satellite longitude [°/E];
- Lt_S the satellite latitude [°/N];
- Lg_n the ES longitude [°/E];
- Lt_n the ES latitude [°/N].

The value of the function Elevation is computed with the following equations:

$$\cos(\theta_n) = \frac{\vec{S}.\vec{S}_n}{\rho.R} = \cos(Lt_S).\cos(Lt_n).\cos(Lg_S - Lg_n) + \sin(Lt_S).\sin(Lt_n) \tag{171}$$

$$\sin(El_n) = \frac{\overrightarrow{S_n S}.\vec{V}_n}{\left\|\overrightarrow{S_n S}\right\|} = \frac{\rho.\cos(\theta_n) - R}{\sqrt{\rho^2 + R^2 - 2.\rho.R.\cos(\theta_n)}} \tag{172}$$

9.6.3 Function Phi_Az()

Phi_Az is the angle [°] of the projection of the considered direction within the plane defined by the antenna main beam axis and its Az-axis, with the antenna main beam axis.

The inputs of the function Phi_ Az (Az_S0, El_S0, i, Az_S, El_S) are:

- Az_S0 the pointed satellite asimuth [°/E];
- El_S0 the pointed satellite elevation [°];
- Az_S the other satellite azimuth [°/E];
- El_S the other satellite elevation [°];
- i the inclination of the antenna Az-axis [°].

The value of the function Phi_ Az is computed with the following equations:

$$x_a = \overrightarrow{S_n S}.\overrightarrow{u_{Az,S_0}} = \begin{pmatrix} +\sin(i).\sin(El_S).\cos(El_{S_0}) \\ +\cos(i).\cos(El_S).\sin(Az_{S_0} - Az_S) \\ -\sin(i).\cos(El_S).\sin(El_{S_0}).\cos(Az_{S_0} - Az_S) \end{pmatrix} \tag{173}$$

$$z_a = \overrightarrow{S_n S}.\overrightarrow{u_{R,S_0}} = \cos(El_{S_0}).\cos(El_S).\cos(Az_{S_0} - Az_S) + \sin(El_{S_0}).\sin(El_S) \tag{174}$$

$$\varphi_{Az} = ArcTan2(x_a, z_a) \tag{175}$$

9.6.4 Function Phi_El()

Phi_El is the angle [°] of the projection of the considered direction within the plane defined by the antenna main beam axis and its El-axis, with the antenna main beam axis.

The inputs of the function Phi_ Az (Az_S0, El_S0, i, Az_S, El_S) are:

- Az_S0 the pointed satellite asimuth [°/E];
- El_S0 the pointed satellite elevation [°];
- Az_S the other satellite azimuth [°/E];
- El_S the other satellite elevation [°];
- i the inclination of the antenna Az-axis [°].

The value of the function Phi_ El is computed with the following equations:

$$y_a = \overrightarrow{S_n S}.\overrightarrow{u_{El,S_0}} = \begin{pmatrix} -\sin(i).\cos(El_S).\sin(Az_{S_0} - Az_S) \\ -\cos(i).\sin(El_{S_0}).\cos(El_S).\cos(Az_{S_0} - Az_S) \\ +\cos(i).\cos(El_{S_0}).\sin(El_S) \end{pmatrix} \quad (176)$$

$$\varphi_{El} = ArcSin(y_a) \quad (177)$$

9.6.5 Functions Phi_cos_Alpha() and Phi_sin_Alpha()

The outputs of these functions are such that:

- Phi_cos_Alpha = Phi * Cos(alpha);
- Phi_sin_Alpha = Phi * sin(alpha);

where:

- Phi: is the off-axis angle [°] of the considered direction measured from the antenna main beam axis;
- Alpha: is the angle [°] between:
  - the plane defined by the antenna main beam axis and the considered direction; and
  - the plane defined by the antenna main beam axis and its Az-axis.

The inputs of these functions Phi_cos_Alpha(Phi_Az, Phi_El) and Phi_sin_Alpha(Phi_Az, Phi_El) are:

- Phi_Az: the angle [°] of the projection of the considered direction within the plane defined by the antenna main beam axis and its Az-axis, with the antenna main beam axis;
- Phi_El: the angle [°] of the projection of the considered direction within the plane defined by the antenna main beam axis and its El-axis, with the antenna main beam axis.

The value of the function Phi_cos_Alpha is computed with the following equations:

$$\varphi_M = ArcCos(z_{a,M}) = ArcCos(\cos(\varphi_{El}).\cos(\varphi_{Az})) \quad (178)$$

$$\alpha_M = ArcTan2(\sin(\varphi_{El}), \cos(\varphi_{El}).\sin(\varphi_{Az})) \quad (179)$$

$$\text{Phi\_cos\_Alpha} = \varphi_M.\cos(\alpha_M) \quad (180)$$

$$\text{Phi\_sin\_Alpha} = \varphi_M.\sin(\alpha_M) \quad (181)$$

9.6.6 Function Inclination_with_Az_El_mount()

The function Inclination_with_Az_El_mount returns the value of the inclination of the antenna Az-axis so that it is aligned with the GSO tangent.

The inputs of the function Az_El_mount(Lg_S, Lt_S, Lg_n, Lt_n) are:

- Lg_S: the pointed satellite longitude [°/E];
- Lt_S: the pointed satellite latitude [°/N];
- Lg_n: the ES longitude [°/E];
- Lt_n: the ES latitude [°/N].

The value of the function Az_El_mount is computed with the set of equations selected in clause 6.9.

9.6.7 Function Inclination_with_E_field_alignment()

Inclination_with_E_field_alignment(Lg_S, Lt_S, Lg_c, Lt_c, i_E0, Lg_n, Lt_n, H_polar).

The function Inclination_with_E_field_alignment returns the value of the inclination of the antenna Az-axis so that it is aligned with the E or H field received from the satellite.

The inputs of the function E_field_alignment(Lg_S, Lt_S, Lg_c, Lt_c, i_E0, Lg_n, Lt_n, H_polar) are:

- Lg_S: the pointed satellite longitude [°/E];
- Lt_S: the pointed satellite latitude [°/N];
- Lg_c: the centre of coverage longitude [°/E];
- Lt_c: the centre of coverage latitude [°/N];
- i_E0: the radiated E field tilt angle [°/N];
- Lg_n: the ES longitude [°/E];
- Lt_n: the ES latitude [°/N];
- H_polar: TRUE when the horizontal polar is used, FALSE when the vertical polar is used.

The value of the function Inclination_with_E_field_alignment is computed with the set of equations selected in clause 6.10.

9.6.8 Function Inclination_with_equatorial_mount()

The function Inclination_with_equatorial_mount returns the value of the inclination of the antenna Az-axis when a perfect equatorial mount is used.

The inputs of the function Inclination_with_equatorial_mount(Lg_S, Lt_S, Lg_n, Lt_n) are:

- Lg_S: the pointed satellite longitude [°/E];
- Lt_S: the pointed satellite latitude [°/N];
- Lg_n: the ES longitude [°/E];
- Lt_n: the ES latitude [°/N].

The value of the function Inclination_with_equatorial_mount is computed with the set of equations selected in clause 6.11.

9.6.9 Function Az_El_mount_alignment_error()

The function returns the maximum value $\delta i_{\max}$ of the offset δi of the antenna azimuth axis $\overrightarrow{u_{Az,S}}$ due to a maximum vertical axis offset $\hat{\theta}_{\max}$, for a antenna with an azimuth-elevation mount.

The inputs of the function Az_El_mount_alignment_error(Vertical_offset, El_S) are:

- Vertical_offset: the maximum vertical axis offset $\hat{\theta}_{\max}$ [°];
- El_S: the elevation of the pointed satellite [°].

The value of the function Az_El_mount_alignment_error is computed with the following equation:

$$\begin{cases} For \quad |El_S| \le \frac{\pi}{2} - |\hat{\theta}_{\max}| & \delta i_{\max} = ArcSin\left(\frac{\sin(|\hat{\theta}_{\max}|)}{\cos(|El_S|)}\right) \\ For \quad \frac{\pi}{2} - |\hat{\theta}_{\max}| \le |El_S| & \delta i_{\max} = \frac{\pi}{2} rad \end{cases} \quad (182)$$

The sign of the Vertical_offset is given to the inclination offset $\delta i_{\max}$:

$$\delta i_{\max} := Sign(\hat{\theta}_{\max}).\delta i_{\max} \quad (183)$$

9.6.10 Function Az_El_mount_with_E_field_alignment_error()

The function returns the value of the offset δi of the antenna azimuth axis $\overrightarrow{u_{Az,S}}$ due to the E or H field alignment error, for an antenna with an azimuth-elevation mount and with polarization alignment.

The input of the function Az_El_mount_with_E_field_alignment_error (di_E) is:

- di_E: the polarization alignment error [°].

The value of the function Az_El_mount_with_E_field_alignment_error is computed with the following equation:

$$\delta i = |\delta i_E| \quad (184)$$

The sign of the di_E is given to the inclination offset δi :

$$\delta i := Sign(\delta i_E).\delta i \quad (185)$$

NOTE: It is recognized that presently the above equations could be simplified (e.g. $\delta i = \delta i_E$) but the general structure of this clause is kept identical to other similar clauses for possible future developments of this clause for taking into consideration the various sources of errors.

9.6.11 Function Equatorial_mount_alignment_error()

The function returns the variation value (Δi) of the antenna Az-axis inclination due to errors on the equatorial mount North-South axis.

The inputs of the function Equatorial_mount_alignment_error(dAz_N, dEl_N, Lt_n) are:

- dAz_N: the North pole direction azimuth offset [°];
- dEl_N: the North pole direction elevation offset [°];
- Lt_n: the ES latitude [°/N].

The value of the function Equatorial_mount_alignment_error is computed with the following equation:

$$\Delta i = 2.ArcSin\left(\sqrt{\sin\left(\frac{\delta El_N}{2}\right)^2 + \sin\left(\frac{\delta Az_N}{2}\right)^2 .\cos\left(Lt_n\right).\cos\left(Lt_n + \delta El_N\right)}\right) \qquad (186)$$

The sign of the dEl_N is given to the inclination offset Δi :

$$\Delta i := Sign\left(\delta El_N\right).\Delta i \qquad (187)$$

9.6.12 Function Max_satellite_longitude_offset()

The function returns the maximum longitude offset $\left(Lg_S - Lg_n\right)$ of a satellite visible at a given minimum elevation.

The inputs of the function Max_satellite_longitude_offset(Lt_n, El_min, Lt_S) are:

- Lt_n: the ES latitude [°/N];
- El_min: the minimum elevation of the satellite [°]. This parameter is optional. The default value is equal to 0°;
- Lt_S: the satellite latitude [°/N]. This parameter is optional. The default value is equal to 0°.

The value of the function Max_satellite_longitude_offset is computed with the following equations:

$$\cos\left(\theta_n\right) = \left(\frac{R}{\rho}\right).\cos\left(El_n\right)^2 + \sin\left(El_n\right).\sqrt{1 - \left(\frac{R}{\rho}\right)^2 .\cos\left(El_n\right)^2} \qquad (188)$$

$$\left(Lg_S - Lg_n\right) = ArcCos\left(\frac{\cos\left(\theta_n\right) - \sin\left(Lt_S\right).\sin\left(Lt_n\right)}{\cos\left(Lt_S\right).\cos\left(Lt_n\right)}\right) \qquad (189)$$

9.6.13 Subroutine GSO_external_contour()

The subroutine GSO_external_contour() computes the contour within the (α, φ) domain of the shadows of the GSO arc and its vicinity for various positions of the ES and the pointed satellite.

This subroutine is activated by the command button "Contour" and by some spin buttons.

The inputs of the subroutine GSO_external_contour() are read in the worksheet "GSO":

- ES_Lg: the earth station longitude [°];
- ES_Lt_min: the earth station minimum latitude [°];
- ES_Lt_max: the earth station maximum latitude [°];
- ES_El_min: the minimum elevation of the pointed satellite [°];
- Hz_El_min: the minimum elevation of the satellite to be protected [°];
- Type_of_antenna_mount: 1 for Az_El_mount_without_alignment;
  2 for Az_El_mount_WITH_alignment;
  3 for Az_El_mount_WITH_polar_alignment;
  4 for Equatorial_mount;
- Permanent_alignment_error: the permanent inclination offset of the antenna Az-axis [°];
- Vertical_offset: the maximum error of the vertical [°] of an Az-El mount;

- dAz_N: the error in azimuth of the pole axis of an equatorial mount [°];
- dEl_N: the error in elevation of the pole axis of an equatorial mount [°].

The contour is computed for three adjacent satellite latitudes (-3°, 0° and +3°), and for different cases:

- the cases where the ES latitude varies from the minimum latitude to the maximum latitude, the ES antenna is successively pointed towards the western and eastern satellites at the minimum elevation, and the satellites to be protected are successively the eastern and western satellites at the minimum elevation;
- the cases where the ES latitude is maximum, the ES antenna is successively pointed towards the western and eastern satellites at the minimum elevation, and the longitude of the satellite to be protected varies from the eastern to the western satellite longitudes at the minimum elevation. If the absolute value of the ES maximum latitude is lower than 1° then computations are made with 1° instead of the ES maximum latitude;
- the cases where the ES latitude is minimum, the ES antenna is successively pointed towards the western and eastern satellites at the minimum elevation , and the longitude of the satellite to be protected varies from the eastern to the western satellite longitudes at the minimum elevation. If the absolute value of the ES minimum latitude is lower than 1° then computations are made with 1° instead of the ES minimum latitude;
- the cases where the ES latitude is minimum, the longitude of the satellite pointed by the ES antenna varies from the eastern to the western satellite longitudes at the minimum elevation, and the longitude of the satellite to be protected is successively the eastern to the western satellite longitudes at the minimum elevation. If the absolute value of the ES minimum latitude is lower than 1° then computations are made with 1° instead of the ES minimum latitude.

In each case, for a given pointed satellite longitude (Lg_S0) and a given longitude (Lg_S) of the other satellite the values of the following variables are computed using the functions already described:

Variables	Equations	Remarks
Az_S0	= Azimuth (Lg_S0, 0, ES_Lg, ES_Lt) '°/N	Azimuth of the pointed satellite
El_S0	= Elevation (Lg_S0, 0, ES_Lg, ES_Lt) '°	Elevation of the pointed satellite
Az_S	= Azimuth (Lg_S, Lt_S, ES_Lg, ES_Lt) '°/N	Azimuth of the other satellite
El_S	= Elevation (Lg_S, Lt_S, ES_Lg, ES_Lt) '°	Elevation of the other satellite
Nominal_inclination	= ... (See next table)	Antenna Az-axis inclination, according to the Type_of_antenna_mount
Alignment_error	= ... (See next table)	Antenna Az-axis inclination error due to the antenna mount axes alignment errors, According to the Type_of_antenna_mount
i_S0	= Nominal_inclination + Alignment_error Permanent_alignment_error	Global inclination of the antenna Az-axis
Phi_Az_S	= Phi_Az (Az_S0, El_S0, i_S0, Az_S, El_S) '°	Off-axis angle along the antenna Az-Axis
Phi_El_S	= Phi_El (Az_S0, El_S0, i_S0, Az_S, El_S) '°	Off-axis angle along the antenna El-Axis
Phi_cos_Alpha_S	= Phi_cos_Alpha (Phi_Az_S, Phi_El_S)	Phi * Cos(Alpha) '° Alpha is defined after the next table
Phi_sin_Alpha_S	= Phi_sin_Alpha (Phi_Az_S, Phi_El_S)	Phi * Sin(Alpha) '° Alpha is defined after the next table

Type_of_antenna_mount	Alignment_error	Nominal_inclination
Az_El_mount_without_alignment	Az_El_mount_alignment_error (Vertical_offset, El_S0)	0
Az_El_mount_WITH_alignment	0	Inclination_with_Az_El_mount (Lg_S0, 0, ES_Lg, ES_Lt)
Az_El_mount_WITH_polar_alignment	0	Inclination_with_E_field_alignment (Lg_S0, 0, Lg_c, Lt_c, i_E0, ES_Lg, ES_Lt, H_polar)
Equatorial_mount	Equatorial_mount_alignment_error (dAz_N, dEl_N, ES_Lt)	Inclination_with_equatorial_mount (Lg_S0, 0, ES_Lg, ES_Lt)

The outputs are stored in the table "GSO_external_contour" of the worksheet "GSO". The outputs are:

- Phi_cos_Alpha_S = Phi * Cos(alpha) '°; and
- Phi_sin_Alpha_S = Phi * Sin(alpha) '°;

where:

- Phi: is the off-axis angle [°] of each considered direction measured from the antenna main beam axis;
- Alpha: is the angle [°] between:
  - the plane defined by the antenna main beam axis and each considered direction; and
  - the plane defined by the antenna main beam axis and its Az-axis.

9.6.14 Subroutine GSO_internal_contour()

The subroutine GSO_internal_contour () computes the minimum off-axis angles within the (α, φ) domain corresponding to the minimum longitude offset (dLg_S_min) for various positions of the ES and the pointed satellite.

This subroutine as the subroutine GSO_internal_contour() is activated by the command button "Contour" and by some spin buttons.

The inputs of the subroutine GSO_internal_contour() are read in the worksheet "GSO". Their are those of the subroutine GSO_external_contour() plus the following input data:

- dLg_S_min: the minimum longitude offset [°].

The minimum off-axis angles corresponding to the minimum longitude offset (dLg_S_min) have been computed for three adjacent satellite latitudes (-3°, 0° and +3°), and for following cases:

- the cases where the ES latitude varies from the minimum latitude to the maximum latitude, the ES antenna is pointed towards the satellite at the ES latitude, and the satellites to be protected are successively the East and West satellites at the minimum longitude offset (dLg_S_min);
- the cases where the ES latitude varies from the minimum latitude to the maximum latitude, the ES antenna is successively pointed towards the West and East satellites at the minimum elevation, and the satellites to be protected are successively the East and West satellites at the minimum longitude offset (dLg_S_min).

The outputs are stored in the table "GSO_internal_contour" of the worksheet "GSO". The structure of that table is identical to the structure of the "GSO_external_contour" table.

9.7 Output data and graphs

9.7.1 General

The outputs of the worksheet "GSO" consist in:

- the GSO shadow for a given ES and a given pointed satellite;
- the GSO_external_contour (See the subroutine GSO_external_contour());
- the GSO_internal_contour (See the subroutine GSO_ internal _contour());
- 4 graphs which have already been presented along the present document.

9.7.2 GSO shadow for a given ES and a given pointed satellite

The GSO shadow on the antenna radiation pattern, within the (α, φ) domain for a given ES and a given pointed satellite uses the following inputs.

Title	Variable		Remarks
Antenna inclination			
i_0		-127,930°	the selected antenna inclination according to the antenna type.
Earth station			
ES latitude	ES_Lt	36,000°/N	∈ [-90°, +90°]
ES longitude	ES_Lg	0,000°/E	∈ [-180°, +180°]
Minimum horizon elevation	Hz_El_min	0,000°	∈ [-90°, +90°], of the other satellite
Pointed satellite			
Azimuth_0	Azimuth_0	101,724	azimuth of the pointed satellite
Elevation_0	Elevation_0	7,000	elevation of the pointed satellite

The GSO shadow on the antenna radiation pattern is computed within the worksheet "GSO" according to the following formulae.

Title	Variable	Formula	Remarks
Satellite latitude	Lt_S	-3	Satellite latitude : - 3, 0 and + 3 successively
Max satellite longitude offset	dLg_S_max	= Max_satellite_longitude_offset (ES_Lt; Hz_El_min; Lt_S)	
Satellite longitude step	dLg_S	= dLg_S_max / 50	
Satellite longitude	Lg_S	= Lg_S + dLg_S	Satellite longitude Lg_S is initiated with : (ES_Lg - dLg_S_max) and is increased up to : (ES_Lg + dLg_S_max)
Az_S - 180°	Az_S_180	= Azimuth (Lg_S; Lt_S; ES_Lg; ES_Lt) - 180	Satellite azimuth
El_S	El_S	= Elevation (Lg_S; Lt_S; ES_Lg; ES_Lt)	Satellite elevation
Phi_Az	Phi_Az	= Phi_Az (Azimuth_0; Elevation_0; i_0; Az_S_180 + 180; Lt_S)	The angle [°] of the projection of the satellite direction within the plane defined by the antenna main beam axis and its Az-axis, with the antenna main beam axis
Phi_El	Phi_El	= Phi_El (Azimuth_0; Elevation_0; i_0; Az_S_180 + 180; El_S)	The angle [°] of the projection of the satellite direction within the plane defined by the antenna main beam axis and its El-axis, with the antenna main beam axis
φ.cos (α)	Phi_cos_Alpha_S	= Phi_cos_Alpha (Phi_Az; Phi_El)	For the meaning of Phi_cos_Alpha_S see the subroutine "GSO_external_contour".
φ.sin (α)	Phi_sin_Alpha_S	= Phi_sin_Alpha (Phi_Az; Phi_El)	For the meaning of Phi_sin_Alpha_S see the subroutine "GSO_external_contour".

History

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